AMERICAN

JOURNAL *of* PHYSICS

Devoted to the Instructional and Cultural Aspects of Physical Science

VOLUME 22   NUMBER 6

SEPTEMBER 1954

AMERICAN ASSOCIATION OF PHYSICS TEACHERS

THE AMERICAN JOURNAL OF PHYSICS is devoted to the instructional and cultural aspects of physical science. Previous to 1940 this publication was known as *The American Physics Teacher*. It is published monthly, except during June, July and August, by the American Institute of Physics for the American Association of Physics Teachers at Prince and Lemon Streets, Lancaster, Pennsylvania.

Manuscripts for publication should be submitted to Dr. Thomas H. Osgood, *American Journal of Physics*, School of Graduate Studies, Michigan State College, East Lansing, Michigan. The authors' institutions are requested to pay an optional publication charge of $4 per page which, when honored, entitles them to 100 free reprints. Instructions will be sent with galley proofs.

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Subscription Price

United States and Canada $6.00
Elsewhere 7.00

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Complete set: Vol. 1, 1933–Vol. 21, 1953—$147.00
Yearly back number rate: $8.00
Single copies: $2.00 each 1933–1936; $1.50 each thereafter.
Cumulative index (Vols. 1–20, 1933–1952): $3.00

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Changes of address should be addressed to the American Institute of Physics.

The contents of the *American Journal of Physics* can be found indexed in the *Education Index*.

Polarization and the Stokes Parameters

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(Received February 8, 1954)

The Stokes parameters have been found to offer a very convenient method for the description of polarization of both electromagnetic radiation and elementary particles. Their development is presented along with some applications to physical problems.

INTRODUCTION

IN the study of both electromagnetic radiation and elementary particles the formalism of the Stokes parameters[1] offers a very convenient tool for the description of polarization. This method shows explicitly the direct analogy between polarization of photons and elementary particles and readily relates this polarization to the polarization of light studied in physical optics courses. It is the purpose of this article to organize into one reference the theory of the Stokes parameters.

In the development of the theory we shall use quantum-mechanical concepts; however, for the application of the Stokes parameters to the polarization of light, no knowledge of quantum mechanics is required. As an introduction we shall first discuss the application of the density matrix (or statistical matrix) to the description of polarization. The Stokes parameters are then defined in terms of elements of the density matrix.

THE DENSITY MATRIX

In quantum mechanics we know that an arbitrary wave equation can be expanded in any desired complete set of orthonormal eigenfunctions. That is,

$$\psi = \sum_i a_i \psi_i. \tag{1}$$

Then

$$|\psi|^* = \sum_{ij} a_i \psi_i a_j^* \psi_j^* = \sum_{ij} a_i a_j^* \psi_i \psi_j^*.$$

From the expansion coefficients we can form a matrix ρ by the rule

$$\rho_{ij} = a_i a_j^*. \tag{2}$$

This matrix is known as the density matrix[2] and has some very useful properties. First we note that $\rho_{ii} = a_i a_i^*$ gives the probability of finding the system in the state characterized by the eigenfunction ψ_i. If we consider the ψ function as being normalized then

$$\int \psi \psi^* d\tau = \sum_{ij} a_i a_j^* \int \psi_i \psi_j^* d\tau = \sum_i a_i a_i^* = \sum_i \rho_{ii} = 1, \tag{3}$$

or

$$\mathrm{Tr}\rho = 1.$$

[1] G. G. Stokes, Trans. Cambridge Phil. Soc. **9**, 399 (1852).

[2] R. C. Tolman, *The Principles of Statistical Mechanics* (Oxford University Press, New York, 1938), p. 325. See also, von Neumann, Nachr. Akad. Wiss. Göttingen, Math.-physik. Kl., p. 245 (1927).

If we make a measurement of some variable F in the system described by the function ψ, the result will be given by

$$\langle F \rangle = \int \psi^* F \psi \, d\tau = \sum_{ij} \int a_i^* \psi_i^* F a_j \psi_j \, d\tau,$$

$$\langle F \rangle = \sum_{ij} a_j a_i^* F_{ij} = \sum_{ij} F_{ij} \rho_{ji},$$

where the matrix F_{ij} is defined by the usual formula

$$F_{ij} = \int \psi_i^* F \psi_j \, d\tau;$$

but

$$\sum_j F_{ij} \rho_{ji} = (F\rho)_{ii}.$$

Therefore,

$$\langle F \rangle = \sum_i (F\rho)_{ii},$$

or

$$\langle F \rangle = \mathrm{Tr}(F\rho), \qquad (4)$$

and since the matrices are all Hermitean, we also have

$$\langle F \rangle = \mathrm{Tr}(\rho F). \qquad (5)$$

If we recall the classical use of density function $\rho(\mathbf{p}, \mathbf{q})$, where the $\mathbf{p}$ and $\mathbf{q}$ are the momentum and position, respectively, which is normalized by

$$\int \rho \, d\tau = 1, \qquad (6)$$

and in terms of which the average value of a variable F is given by

$$\langle F \rangle = \int F \rho \, d\tau, \qquad (7)$$

then we see immediately the similar role played by the density matrix in quantum mechanics from a comparison of Eqs. (3) and (5) with (6) and (7).

Polarization of electromagnetic radiation is usually described by the vibration of the electric vector. For a complete description, it can be shown[3] that a plane wave may be thought of as being composed of two physically independent (incoherent) beams of orthogonal polarization. That is, the electric vector may be analyzed by

[3] See, for example, L. Landau and E. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley Press, Cambridge, 1951), pp. 126–129.

the equation

$$\mathbf{E} = a_1 \mathbf{E}_1 + a_2 \mathbf{E}_2, \qquad (8)$$

where $\mathbf{E}_1$ and $\mathbf{E}_2$ are two orthogonal unit vectors, and the a_i, which in general are complex, describe the amplitude and phase of the two vibrations. From the two expansion coefficients we can form a 2×2 density matrix.

Polarization of particles is described by the orientation of the spin. For the case of particles of spin $1/2$, we need only two orthonormal wave functions to form a complete set describing the polarization of the particle.[4] Hence

$$\psi = a_1 \Phi_1 + a_2 \Phi_2, \qquad (9)$$

where Φ_1 is the eigenfunction for the spin quantum number $+1/2$ and Φ_2 is that for $-1/2$. Again the expansion coefficients form a 2×2 density matrix.

The similarity of Eqs. (8) and (9) immediately suggest that the description of polarization of electromagnetic radiation and of particles will be similar.[5] In fact we can use the same equation for the description of both[6] if we write

$$\psi = a_1 \psi_1 + a_2 \psi_2, \qquad (10)$$

where we must distinguish two cases.[7]

Case I. Electromagnetic Radiation or Photons

The ψ_1 and ψ_2 can either represent two orthogonal states of plane polarization or two states of circular polarization. Then $|a_1|^2$ and $|a_2|^2$ give the probabilities of detection of quanta by a detector sensitive only to states ψ_1 and ψ_2, respectively.

Case II. Particles of Spin 1/2

The wave functions ψ_1 and ψ_2 can represent either of two opposite spin orientations, parallel and antiparallel to the momentum. Again the $|a_i|^2$ give the probabilities of detection of particles specified by the eigenfunctions ψ_i.

[4] See, for example, L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), p. 224.

[5] See also, P. Jordan, Z. Physik **44**, 292 (1927).

[6] D. L. Falkoff and J. E. MacDonald, J. Opt. Soc. Am. **41**, 861 (1951).

[7] See, for example, H. A. Tolhoek and S. R. DeGroot, Physica **17**, 1 (1951).

In both cases the density matrix

$$\rho = \begin{pmatrix} a_1 a_1{}^* & a_1 a_2{}^* \\ a_2 a_1{}^* & a_2 a_2{}^* \end{pmatrix} \qquad (11)$$

completely characterizes the beam since we can obtain the intensities of the two polarization states from the diagonal elements while the off-diagonal elements furnish the relative phase.

Complete polarization can be represented by a single eigenfunction

or

$$\psi = a_1 \psi_1 \quad \text{with} \quad \rho_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$
$$\psi = a_2 \psi_2 \quad \text{and} \quad \rho_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \qquad (12)$$

where the ψ_i refer to a state of pure polarization and we are considering a beam of unit intensity.

Since an unpolarized beam may be considered as the incoherent superposition of two polarized beams with equal intensity, to get the density matrix for an unpolarized beam we add those of the two polarized beams

$$\rho_u = 1/2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \qquad (13)$$

the 1/2 appears because of the normalization by Eq. (3).

This may be more apparent if we use the equation

$$\mathbf{E} = \cos\theta e^{i\Phi} \mathbf{E}_1 + \sin\theta \mathbf{E}_1$$

to represent the beam. The density matrix is then

$$\begin{pmatrix} \cos^2\theta & \cos\theta \sin\theta e^{i\Phi} \\ \sin\theta \cos\theta e^{-i\Phi} & \sin^2\theta \end{pmatrix}.$$

For an unpolarized beam we must average over the angles θ and Φ. Hence,

$$\rho_u = \begin{pmatrix} \tfrac{1}{2} & 0 \\ 0 & \tfrac{1}{2} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

In general, a beam will have an arbitrary degree of polarization and we can characterize such a beam by the incoherent superposition of an unpolarized beam and a totally polarized one. If the polarized portion is described by Eq. (9), then its contribution to the density matrix will be given by Eq. (11). The unpolarized portion will be characterized by Eq. (13) so that the density matrix for an arbitrary beam can be written in the form

$$\rho = U \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + P \begin{pmatrix} a_1 a_1{}^* & a_1 a_2{}^* \\ a_2 a_1{}^* & a_2 a_2{}^* \end{pmatrix}, \quad (14)$$

where P represents the degree of polarization. P is real and $0 \leq P \leq 1$. We now note the following three cases:

(1) If $0 < P < 1$, then the beam is partially polarized.

(2) If $P = 0$, then the beam is unpolarized.

(3) If $P = 1$, then the beam is totally polarized.

For the case $P = 0$, we know that

$$\rho = 1/2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$$

thus $U = 1/2$ when $P = 0$.

For the case $P = 1$, the density matrix is given by Eq. (11) so that

$$U = 0 \quad \text{when} \quad P = 1.$$

These conditions are satisfied by $U = 1/2 \times (1 - P)$ so that for the general case, the density matrix is given by

$$\rho = 1/2(1 - P) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + P \begin{pmatrix} a_1 a_1{}^* & a_1 a_2{}^* \\ a_2 a_1{}^* & a_2 a_2{}^* \end{pmatrix}. \quad (15)$$

By the proper choice of pure states of polarization ψ_i, the part of the density matrix representing total polarization can be written in one of the forms given by Eq. (12); therefore, we may write the general density matrix as

or

$$\rho = 1/2(1 - P) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + P \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$
$$\rho = 1/2 \begin{pmatrix} 1 + P & 0 \\ 0 & 1 - P \end{pmatrix}. \quad (16)$$

Hence any intensity measurement made in relation to these pure states will yield the eigenvalues

$$I_1 = 1/2(1 + P),$$
$$I_2 = 1/2(1 - P). \qquad (17)$$

We note that the definition of P is independent of the choice of pure states and gives only the degree of mixture of polarized and unpolarized light. We shall now introduce a description which is dependent upon the type of polarization.

THE STOKES PARAMETERS FOR ELECTROMAGNETIC RADIATION

To determine experimentally the state of polarization of an arbitrary beam of electromagnetic radiation (photons) we must make a set of four measurements. These measurements can best be explained using the analogy of optics where Nicol prisms and quarter-wave plates are used. However, it must be remembered that these devices will not work for high-energy photons and particles, but that other analogous experiments must be used. These analogous experiments will be discussed in a later section. The most convenient set of four measurements are those that yield the following information:

(1) The intensity of the beam.
(2) The degree of plane polarization with respect to two arbitrary orthogonal axes.
(3) The degree of plane polarization with respect to a set of axes oriented at 45° to the right of the previous one.
(4) The degree of circular polarization.

In optics the second and third of these measurements can be made with a Nicol prism while the fourth requires the additional use of a quarter-wave plate.

Let us now consider these measurements in terms of Eq. (10):

$$\psi = a_1\psi_1 + a_2\psi_2.$$

Throughout this discussion we shall consider the beam to be normalized to unit intensity so that an intensity measurement will yield

$$I = a_1 a_1{}^* + a_2 a_2{}^* = 1. \quad (18)$$

In terms of the states described by ψ_1 and ψ_2 we shall now define an orientation coefficient

$$P(\psi_1,\psi_2) = a_1 a_1{}^* - a_2 a_2{}^* = \rho_{11} - \rho_{22}, \quad (19)$$

which gives the difference of the intensity measurements of the pure states defined by ψ_1 and ψ_2.

So far the ψ_i have not been chosen and can refer equally well to plane polarization or circular polarization states.

The general equation for polarization in terms of the **E** vector is

$$\mathbf{E} = b_1 \exp[i(\omega t + \delta_1)]\mathbf{E}_1 + b_2 \exp[i(\omega t + \delta_2)]\mathbf{E}_2, \quad (20)$$

where the b_i are real, and the $\mathbf{E}_i$ are orthogonal unit vectors chosen arbitrarily in the plane orthogonal to the direction of propagation. From this equation we note the following cases:

(1) If the phase difference $\varphi = \delta_1 - \delta_2 = 0$ we have plane-polarized radiation.
(2) If $b_1 = b_2$ and $\varphi = \pm\pi/2$, we have right or left circular polarization depending upon whether the sign is $+$ or $-$, respectively.
(3) If $b_1 \neq b_2 \neq 0$ and $\varphi \neq 0$, we have elliptical polarization.

For the first polarization measurement we measure the intensities transmitted by a Nicol prism oriented in the two directions $\mathbf{E}_1$ and $\mathbf{E}_2$. Thus we have chosen

$$\psi_1 = \mathbf{E}_1 \quad \text{and} \quad \psi_2 = \mathbf{E}_2.$$

The first orientation coefficient will then be

$$P(\psi_1,\psi_2) = P_1 = a_1 a_1{}^* - a_2 a_2{}^*$$

or

$$P_1 = \rho_{11} - \rho_{22}. \quad (21)$$

In terms of Eq. (20) this is $P_1 = b_1{}^2 - b_2{}^2$.

The second measurement is similar to the first but oriented at 45° to the right. With this choice of axes

$$\psi = a_1'\psi_1' + a_2'\psi_2',$$

where

$$\psi_1' = \mathbf{E}_1 \cos 45° + \mathbf{E}_2 \sin 45°,$$
$$\psi_2' = -\mathbf{E}_1 \sin 45° + \mathbf{E}_2 \cos 45°,$$

and

$$P(\psi_1',\psi_2') = P_2 = a_1'a_1'{}^* - a_2'a_2'{}^*,$$

or

$$P_2 = \rho_{11}' - \rho_{22}'.$$

Since we have the same beam,

$$\psi = a_1'\psi_1' + a_2'\psi_2' = a_1\psi_1 + a_2\psi_2.$$

From Eq. (22) we have

$$\psi_1 = (1/\sqrt{2})\psi_1' - (1/\sqrt{2})\psi_2',$$
$$\psi_2 = (1/\sqrt{2})\psi_1' + (1/\sqrt{2})\psi_2',$$

or

$$a_1'\psi_1' + a_2'\psi_2'$$
$$= (1/\sqrt2)a_1(\psi_1' - \psi_2') + (1/\sqrt2)a_2(\psi_1' + \psi_2'),$$
$$= (1/\sqrt2)(a_1 + a_2)\psi_1' + (1/\sqrt2)(a_2 - a_1)\psi_2'.$$

Thus

$$a_1' = (1/\sqrt2)(a_1 + a_2), \quad \text{and} \quad a_2' = (1/\sqrt2)(a_2 - a_1).$$

From which, after a simple multiplication, we find

$$P_2 = \rho_{12} + \rho_{21}. \tag{22}$$

In terms of Eq. (20) this is readily seen to be

$$P_2 = 2b_1 b_2 \cos(\delta_1 - \delta_2).$$

The third measurement is one for circularly polarized light. To make the measurement we insert a quarter-wave plate with its fast axis 45° to the right of $\mathbf{E}_1$ and make intensity measurements with the transmission axis of the Nicol prism oriented along $\mathbf{E}_1$ and $\mathbf{E}_2$. That is, we are making the choice

$$\psi_R = (1/\sqrt2)e^{i\varphi}\psi_1 + (1/\sqrt2)\psi_2,$$

where $\varphi = \frac12 2\pi$, or $e^{i\pi/2} = i$, and

$$\psi_L = (1/\sqrt2)\psi_1 + (1/\sqrt2)e^{i\varphi}\psi_2,$$

with

$$P(\psi_R, \psi_L) = \rho_{11}^c - \rho_{22}^c. \tag{23}$$

Thus

$$\psi_1 = (1/\sqrt2)(\psi_L - i\psi_R) \left.\right\} \tag{24}$$
$$\psi_2 = (1/\sqrt2)(\psi_R - i\psi_L)$$

then since $\psi = a_1\psi_1 + a_2\psi_2$, we have

$$\psi = (a_1/\sqrt2)(\psi_L - i\psi_R) + (a_2/\sqrt2)(\psi_R - \psi_L),$$
$$\psi = (1/\sqrt2)(a_2 - ia_1)\psi_R + (1/\sqrt2)(a_1 - ia_2)\psi_L$$
$$= a_1^c\psi_R + a_2^c\psi_L. \tag{25}$$

Substituting the values of a_i^c from Eq. (25) in Eq. (23) we find

$$P(\psi_R, \psi_L) = P_3 = i(\rho_{21} - \rho_{12}). \tag{26}$$

In terms of Eq. (20) this is $2b_1 b_2 \sin(\delta_1 - \delta_2)$. The appearance of the imaginary number i in P_3 is the mathematical expression of the fact that we must use a quarter-wave plate since a Nicol prism by itself is not sensitive to circular polarization.

As a result of these four measurements, we have a set of four quantities:

$$\left.\begin{array}{l} I = \rho_{11} + \rho_{22} \\ P_1 = \rho_{11} - \rho_{22} \\ P_2 = \rho_{12} + \rho_{21} \\ P_3 = i(\rho_{21} - \rho_{12}) \end{array}\right\}, \tag{27}$$

which are known as the Stokes parameters and completely characterize a beam of electromagnetic radiation. For pure states of polarization a measurement of the Stokes parameters can easily be seen to provide the following results:

$P_1 = +1$; plane polarization along $\mathbf{E}_1$
$P_1 = -1$; plane polarization along $\mathbf{E}_2$
$P_2 = \pm1$; plane polarization at an angle of 45° to the right of $\mathbf{E}_1$ and $\mathbf{E}_2$, respectively,
$P_3 = +1$; right circular polarization
$P_3 = -1$; left circular polarization.

From the Stokes parameters we can readily construct the density matrix

$$\rho = 1/2\begin{pmatrix} 1 + P_1 & P_2 + iP_3 \\ P_2 - iP_3 & 1 - P_1 \end{pmatrix}. \tag{28}$$

THE STOKES PARAMETERS OF AN ELECTRON

In this section we will follow the development given by Tolhoek and De Groot.[7] The general state of a beam of electrons can again be described by

$$\psi = a_1\psi_1 + a_2\psi_2, \tag{10}$$

where ψ_1 represents a state of spin $+1/2$ in the positive z direction, ψ_2 represents a state of spin $-1/2$ in the positive z direction, and the spin direction is defined in the rest system of the electron. It can be shown[8] that an arbitrary spin direction in relation to the z axis is given by

$$a_2/a_1 = \tan\tfrac12\theta e^{i\varphi}, \tag{29}$$

where the angles θ and φ give the orientation of the spin as shown in Fig. 1. If an arbitrary phase
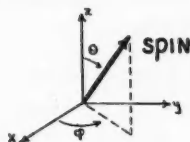

Fig. 1. Angles θ and ϕ give the spin orientation with respect to the axes.

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[8] P. A. M. Dirac, *The Principles of Quantum Mechanics* (Oxford University Press, New York, 1930), p. 136.

is neglected, we can choose

$$a_1 = \cos 1/2\theta; \quad a_2 = \sin 1/2\theta e^{i\varphi}. \quad (30)$$

Then the state of polarization in the x direction will be given by the values

$$\psi_1' \begin{cases} \theta = \pi/2 \\ \varphi = 0 \end{cases}; \quad \psi_2' \begin{cases} \theta = \pi/2 \\ \varphi = \pi \end{cases}.$$

Therefore

$$\psi_1' = (1/\sqrt{2})(\psi_1 + \psi_2)$$

and

$$\psi_2' = (1/\sqrt{2})(\psi_1 - \psi_2); \quad (31)$$

and for states of polarization in the y direction, designated by the functions ψ_1'' and ψ_2'', we have:

$$\psi_1'' \begin{cases} \theta = \pi/2 \\ \varphi = \pi/2 \end{cases} \quad \text{and} \quad \psi_2'' \begin{cases} \theta = \pi/2 \\ \varphi = \pi/2 \end{cases}.$$

Therefore

$$\psi_1'' = (1/\sqrt{2})(\psi_1 + i\psi_2);$$
$$\psi_2'' = (1/\sqrt{2})(\psi_1 - i\psi_2). \quad (32)$$

Let us again consider a beam of electrons normalized to unit intensity so that

$$I = a_1 a_1^* + a_2 a_2^* = 1.$$

If we investigate a beam of electrons by taking a measurement to determine the number of spins in the z direction with the value $+1/2$ (which yields $I_1 = a_1 a_1^*$) and another measurement for $-1/2$ (which yields $I_2 = a_2 a_2^*$), then we obtain the orientation coefficient.

$$P_1 = a_1 a_1^* - a_2 a_2^* = \rho_{11} - \rho_{22}. \quad (33)$$

To investigate a spin state at right angles to this one in the x direction, we have from Eq. (31),

$$\psi_1 = (1/\sqrt{2})(\psi_1' + \psi_2'); \psi_2 = (1/\sqrt{2})(\psi_1' - \psi_2'). \quad (34)$$

Upon substitution of Eq. (34) into Eq. (10),

$$\psi = (a_1/\sqrt{2})(\psi_1' + \psi_2') + (a_2/\sqrt{2})(\psi_1' - \psi_2')$$
$$= (1/\sqrt{2})(a_1 + a_2)\psi_1 + (1/\sqrt{2})(a_1 - a_2)\psi_2.$$

Then

$$P_2 = \rho_{11}' - \rho_{22}' = (1/2)(a_1 + a_2)(a_1^* + a_2^*)$$
$$- (1/2)(a_1 - a_2)(a_1^* - a_2^*)$$
$$= a_1 a_2^* + a_2 a_1^*.$$

Therefore,

$$P_2 = \rho_{12} + \rho_{21}. \quad (35)$$

TABLE I. Meanings of Stokes parameters.

Stokes parameter	Photon observation	Electron observation
I	Intensity	Intensity
P_1	Plane polarization	Transverse spin
P_2	Plane polarization at an angle of $\pi/4$ to the previous direction	Transverse spin at an angle of $\pi/2$ to the previous direction
P_3	Circular polarization	Longitudinal spin

For states polarized in the y direction, from Eq. (32):

$$\psi_1 = (1/\sqrt{2})(\psi_1'' + \psi_2'')$$

and

$$\psi_2 = (1/\sqrt{2})(-i\psi_1'' + i\psi_2'').$$

A similar calculation leads to

$$P_3 = i(\rho_{12} - \rho_{21}). \quad (36)$$

Thus the four measured quantities, again called the Stokes parameters, are given by

$$\left. \begin{array}{l} I = \rho_{11} + \rho_{22} \\ P_1 = \rho_{11} - \rho_{22} \\ P_2 = \rho_{12} + \rho_{21} \\ P_3 = i(\rho_{21} - \rho_{12}) \end{array} \right\}, \quad (37)$$

and we notice that Eqs. (37) are identical to Eqs. (27).

Scattering experiments are generally used to determine the Stokes parameters of electrons; Tolhoek and DeGroot[7] show in their article that scattering experiments are not sensitive to longitudinal spin, so that if we desire to make a measurement of longitudinal spin we must first change it to transverse spin. This reminds us of the fact that in optics a Nicol prism is not sensitive to circular polarization and that the circular polarization must be transformed to plane polarization by means of a quarter-wave plate before a measurement can be made. Thus if we choose single scattering as the electron polarization detector as being analogous to the Nicol prism, then we can attach the meanings to the Stokes parameters as shown in Table I.

This description for electrons can readily be extended to all elementary particles of spin $\pm 1/2$.

PROPERTIES OF THE STOKES PARAMETERS

In the application of the Stokes parameters to any problem it is customary to write them in the

form of a four vector.

$$\begin{bmatrix} I \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} I \\ \mathbf{P} \end{bmatrix}. \tag{38}$$

As an example of its use, consider the following simple cases:

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$ represents an unpolarized beam;

$$\begin{bmatrix} 1 \\ \pm 1 \\ 0 \\ 0 \end{bmatrix}$$ and $$\begin{bmatrix} 1 \\ 0 \\ \pm 1 \\ 0 \end{bmatrix}$$ represent plane polarization or transverse spin;

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ \pm 1 \end{bmatrix}$$ represent circular polarization or longitudinal spin.

Since the Stokes parameters are dependent upon the choice of axes, there must exist a transformation matrix M which will relate the Stokes parameters in one coordinate system to another system. If we consider a second coordinate system rotated about the direction of propagation an angle θ to the right of the original coordinate system, then

$$\begin{pmatrix} I' \\ \mathbf{P}' \end{pmatrix} = M \begin{pmatrix} I \\ \mathbf{P} \end{pmatrix}, \tag{39}$$

where

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{40}$$

Thus if in the old system we had (I, P_1, P_2, P_3), then in the new system of coordinates, the Stokes parameters of the same beam would be

$$\begin{bmatrix} I \\ P_1 \cos 2\theta + P_2 \sin 2\theta \\ -P_1 \sin 2\theta + P_2 \cos 2\theta \\ P_3 \end{bmatrix}. \tag{41}$$

A rotation in the opposite direction changes the sign of the $\sin 2\theta$ terms.

Next let us consider a partially polarized beam and determine the probability of detecting a given polarization in it. First let us consider the probability of finding a photon or an electron in the states described by ψ_1 and ψ_2. For a normalized beam

$$I = a_1 a_1^* + a_2 a_2^* = 1 \quad \text{and} \quad P_1 = a_1 a_1^* - a_2 a_2^*.$$

Then by subtraction and addition we find, respectively,

$$a_1 a_1^* = (1/2)(1 + P_1)$$

and

$$a_2 a_2^* = (1/2)(1 - P_1). \tag{42}$$

Since the state represented by ψ_1 is characterized by the Stokes parameters (1 1 0 0) and that of ψ_2 by (1 −1 0 0), we can get the same probability for detection in an arbitrary beam characterized by $(I \, P_1 \, P_2 \, P_3)$ from the simple vector multiplications

$$(1/2)(1 \ 1 \ 0 \ 0) \begin{bmatrix} I \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = (1/2)(1 + P_1);$$

and

$$(1/2)(1 \ -1 \ 0 \ 0) \begin{bmatrix} I \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = (1/2)(1 - P_1).$$

In a similar manner it can easily be shown that the probabilities of detection of polarization in the other two states of polarization are given by

$$(1/2)(1 \pm P_2) \quad \text{and} \quad (1/2)(1 \pm P_3). \tag{43}$$

Now it can readily be verified by a simple calculation that these results can be expressed in two forms which are useful in the application of the theory.

(1) Let w be the probability of detecting a photon or a particle characterized by the density matrix ρ' in an arbitrary beam characterized by the density matrix ρ. Then

$$w = \mathrm{Tr}(\rho \rho'), \tag{44}$$

where we note the similarity of Eq. (44) to Eq. (4).

(2) In terms of the Stokes parameters, let us determine the probability of detecting a photon or a particle characterized by the Stokes parameters $(1, \mathbf{Q})$ in an arbitrary beam characterized by the Stokes parameters $(I, \mathbf{P})$. For this we use an analyzer which will pass only states of polarization $(1, \mathbf{Q})$ then

$$w = (1/2)(1 + \mathbf{P} \cdot \mathbf{Q}) = (1/2)(1, \mathbf{Q}) \begin{pmatrix} I \\ \mathbf{P} \end{pmatrix}. \tag{45}$$
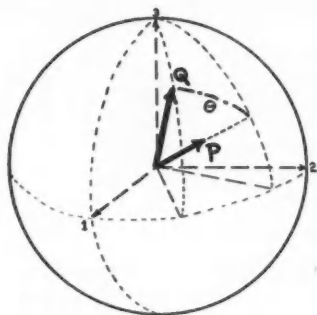
FIG. 2. Poincaré sphere representation of the orientation coefficients. $\mathbf{Q}$ is a unit vector representing the analyzer setting, while $\mathbf{P}$ is a vector $(0 \leq |P| \leq 1)$ representing the state of polarization of the beam.

With this last notation we may map[9] the orientation coefficients on the Poincaré sphere of radius 1 as shown in Fig. 2. In this representation, the three components P_1, P_2, and P_3 are orthogonal. With respect to the polarization axes specified by 1, 2, 3 in the figure, the polarization of the beam will be given by a vector $\mathbf{P}(0 \leq |\mathbf{P}| \leq 1)$ oriented in some given direction. The different settings of an analyzer used to determine the polarization will be mapped by a unit vector $\mathbf{Q}$. Then from the figure we see that $\mathbf{P} \cdot \mathbf{Q}$ is given by $P \cos\theta$.

The Stokes parameters also have the property that, if several independent beams are superposed incoherently, then the Stokes parameters of the resulting beam are just the sum of the parameters characterizing the individual beams.[10] That is, for the final beam, we have

$$
\begin{aligned}
I &= \sum_i I^{(i)}, \\
P_1 &= \sum_i P_1^{(i)}, \\
\times \quad P_2 &= \sum_i P_2^{(i)}, \\
P_3 &= \sum_i P_3^{(i)}.
\end{aligned}
\tag{46}
$$

DESCRIPTION OF INTERACTIONS

If photons or particles undergo an interaction which is sensitive to polarization then, in general, the Stokes parameters of the initial beam will be transformed into a new set of parameters. The relations between these two Stokes vectors is given by a matrix T characteristic of the inter-

[9] U. Fano, J. Opt. Soc. Am. 39, 859 (1949).
[10] S. Chandrasekhar, Astrophys. J. 105, 424 (1947).

action. That is,

$$
\begin{pmatrix} I \\ \mathbf{P} \end{pmatrix} = T \begin{pmatrix} I_0 \\ \mathbf{P}_0 \end{pmatrix},
\tag{47}
$$

where T is a 4×4 array of numbers. Unfortunately, the interaction matrices have not been worked out for all polarization-sensitive interactions. In this section we shall discuss a few of those that have been worked out to provide an example of the ease of description in terms of the Stokes parameters.

When we pass a beam $(I, \mathbf{P})$ through a polarizer T and detect it with an analyzer $(1, \mathbf{Q})$, then the fractional intensity detected is given by

$$
w = (1/2)(1, \mathbf{Q}) T \begin{pmatrix} I \\ \mathbf{P} \end{pmatrix}.
\tag{48}
$$

Since polarization is first studied in the realm of optics, let us start with some examples applied to this field.

(1) A Nicol prism with its transmission axis along $\mathbf{E}_1$, i.e., one which will only pass light characterized by the Stokes parameters (1 1 0 0). In this case the interaction matrix is

$$
T = (1/2) \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.
\tag{49}
$$

This matrix expresses the $\cos^2\Phi$ dependence on the orientation of the electric vector as can be seen from the following considerations. For the vector expression $\mathbf{E} = \cos\Phi \mathbf{E}_1 + \sin\Phi \mathbf{E}_2$ we have the Stokes vector

$$
\begin{bmatrix} 1 \\ \cos^2\Phi - \sin^2\Phi \\ 2\sin\Phi\cos\Phi \\ 0 \end{bmatrix},
$$

and hence

$$
(1/2) \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \cos^2\Phi - \sin^2\Phi \\ 2\sin\Phi\cos\Phi \\ 0 \end{bmatrix}
$$

$$
= (1/2) \begin{bmatrix} 1 + \cos^2\Phi - \sin^2\Phi \\ 1 + \cos^2\Phi - \sin^2\Phi \\ 0 \\ 0 \end{bmatrix} = \cos^2\Phi \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}.
$$

Now consider that we have two Nicol prisms. The first acts as a polarizer and has the interaction matrix T; the second we use as an analyzer which accepts polarization characterized by $(1, \mathbf{Q})$. Then the intensity accepted by the analyzer after the light has passed through the polarizer is given by Eq. (48).

If the initial beam is unpolarized and the transmission axes of the Nicol prisms are parallel, then

$$w = (1/2)(1 \ 1 \ 0 \ 0) T \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= (1/2)(1 \ 1 \ 0 \ 0)(1/2) \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = 1/2,$$

i.e., only half the intensity of the original beam is transmitted. For crossed Nicols

$$w = (1/2)(1 \ -1 \ 0 \ 0) T \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = 0,$$

i.e., no light is passed. Other more complicated combinations can readily be worked out, where it must be remembered that the Stokes parameters of the initial beam must be given so that $P_1 = 1$ refers to plane polarization along the transmission axis of the Nicol prism.

The next two examples are among those worked out by Perrin[11] in his article on the scattering of light.

(2) A birefringent crystal which introduces a phase φ between the components of the vibration along two orthogonal axes. If we take the fast axis along $\mathbf{E}_1$, then

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\varphi & -\sin\varphi \\ 0 & 0 & \sin\varphi & \cos\varphi \end{pmatrix}. \quad (50)$$

In particular, a quarter-wave plate with its fast axis along $\mathbf{E}_1$ is given by

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \quad (51)$$

[11] F. Perrin, J. Chem. Phys. 10, 415 (1942).

As an example, let us consider the conversion of circularly polarized light into plane polarized light.

$$T \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix},$$

which says that right circularly polarized light passing through a quarter-wave plate with its fast axis along $\mathbf{E}_1$ is converted into plane-polarized light oriented 45° to the right of $\mathbf{E}_2$. The inverse effect is given by

$$T \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix},$$

where light polarized 45° to the right of $\mathbf{E}_1$ is converted into right circularly polarized light.

(3) A crystal exhibiting optical activity which rotates the plane of polarization an angle φ to the right has the interaction matrix

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\varphi & -\sin 2\varphi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (52)$$

These examples demonstrate the simplicity of treating polarized light in the formalism of the Stokes parameters.

Next let us consider the polarization of gamma rays. In the Compton scattering of photons the Klein-Nishina formula shows that the cross section depends on the angle between the directions of polarization of the incident and scattered photon. Fano[9] has shown that the treatment of Compton scattering in terms of the Stokes parameters greatly facilitates the computation of the various cross sections. For our examples in this discussion we shall only discuss plane polarization effects, in which case the interaction matrix developed by Fano reduces to a 3×3 matrix expressing the usual Klein-Nishina cross section:

$$T = (1/2)(e^2/mc^2)(k/k_0)^2$$
$$\times \begin{pmatrix} 1+\cos^2\theta+(k_0-k)(1-\cos\theta) & -\sin^2\theta & 0 \\ -\sin^2\theta & 1+\cos^2\theta & 0 \\ 0 & 0 & 2\cos\theta \end{pmatrix},$$
$$(53)$$

where k_0 and k are the energy of the incident and scattered quanta, respectively, in units of mc^2, and θ is the angle of scattering between k_0 and k. With this matrix, positive values of the Stokes parameters have the following significance:

P_1 represents plane polarization in the plane of scattering.

P_2 represents plane polarization in a plane 45° to the right.

The complete 4×4 matrix is given in Fano's article. The fourth row and column, omitted here, contain terms dependent upon the spin orientation of the scattering electron. Using Fano's interaction matrix, the Compton cross section per unit solid angle is then given by

$$\frac{d\sigma}{d\Omega} = (1/2)(1,\mathbf{Q})T\binom{I}{\mathbf{P}}. \qquad (54)$$

As examples of its use we shall discuss three cases.

(1) Compton Scattering of Unpolarized Gamma Rays

As a result of Compton scattering the Stokes parameters of an unpolarized beam undergo the transformation

$$T\begin{pmatrix}1\\0\\0\end{pmatrix} \sim \begin{pmatrix}1+\cos^2\theta+(k_0-k)(1-\cos\theta)\\-\sin^2\theta\\0\end{pmatrix}, \qquad (55)$$

from which, since $P_1 \sim -\sin\theta$, we see that as a result of Compton scattering the beam is partially polarized orthogonal to the plane of scattering. The degree of polarization is usually defined as

$$p = d\sigma_\perp/d\sigma_\|, \quad \text{where} \quad d\sigma_\perp = (1/2)(1 \quad -1 \quad 0)T\begin{pmatrix}1\\0\\0\end{pmatrix},$$

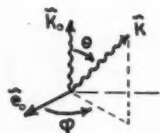$$d\sigma_\| = (1/2)(1 \quad 1 \quad 0)T\begin{pmatrix}1\\0\\0\end{pmatrix},$$

FIG. 3. Compton scattering of a polarized gamma-ray beam. $\mathbf{e_0}$ gives the plane of polarization of the incident beam $\mathbf{k_0}$, which is oriented at an angle ϕ to the plane of scattering. θ is the Compton scattering angle.

yielding the result

$$p = \frac{(k_0-k)(1-\cos\theta)+2}{(k_0-k)(1-\cos\theta)+2\cos^2\theta},$$

where the $\perp$ and $\|$ refer to plane polarization perpendicular to and parallel to the plane of scattering, respectively. Using the ordinary Klein-Nishina formula the derivation of this result requires much more effort and careful consideration of the angles involved. Thus in analogy to optics we see that the polarization of gamma rays by Compton scattering is very similar to the polarization of light by reflection from a dielectric.

(2) Compton Scattering of a Polarized Beam

For this discussion we shall use the geometry shown in Fig. 3. To use Fano's matrix we must recall that positive values of P_1 refer to plane polarization in the plane of scattering. The simplest representation of the initial beam is (1 1 0), but this is in a coordinate system rotated through an angle Φ to the right of the plane of scattering (looking in the direction $-\mathbf{k_0}$); therefore, we must rotate the coordinate system an angle Φ to the left using the matrix M given by Eq. (40) with the appropriate change of sign. Thus, using the plane of scattering as a reference plane, we have

$$M\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}1\\\cos2\Phi\\\sin2\Phi\end{pmatrix} = \begin{pmatrix}1\\\cos^2\Phi-\sin^2\Phi\\2\sin\Phi\cos\Phi\end{pmatrix},$$

which yields the final result

$$d\sigma/d\Omega = (1/2)(1 \ 0 \ 0)T\begin{pmatrix}1\\\cos^2\Phi-\sin^2\Phi\\2\sin\Phi\cos\Phi\end{pmatrix}$$

$$\sim k_0/k+k/k_0-2\sin^2\theta\cos^2\Phi,$$

the usual result, but found in a much simpler manner. Here we have used (1 0 0) as an analyzer since we are interested in the intensity of the scattered beam regardless of its polarization. This is just the Stokes vector characterizing an ordinary photon detector, such as a scintillation counter. Thus we see that the polarized gamma rays are preferentially scattered in a direction perpendicular to the electric vector. This allows
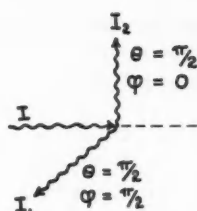
Fig. 4. Compton scattering as an analyzer for plane polarization such that for $P_1 = 1$ the beam is plane polarized in the (I, I_2) plane. The degree of polarization is given by

$$I_2/I_1 = (p+R)/(pR+1),$$

where R is the ratio that would be obtained for a totally polarized beam.

us to use Compton scattering as an analyzer in a way analogous to the Nicol prism of optics.

To use Compton scattering as an analyzer for plane polarization, the two measurements shown in Fig. 4 are made. The partially polarized beam is Compton scattered and intensity measurements are made for a scattering angle of $\theta = \pi/2$ in the two directions I_2 and I_1, where I_1 is perpendicular to the plane of I and I_2. The degree of polarization p with respect to these two directions is found from $I_2/I_1 = (p+R)/(pR+1)$, where R is the ratio that would be obtained for a beam totally polarized in the (I, I_2) plane.

(3) The Double Compton Scattering Experiment, Shown in Fig. 5

The cross section for double scattering is given by

$$d\sigma/d\Omega = (1/2)(1\ 0\ 0)T_2 T_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

The unpolarized incident beam, $\mathbf{k}_0$, characterized by $(1\ 0\ 0)$, will be partially polarized as a result of the first scattering. After this first scattering, the beam $\mathbf{k}_1$ will be characterized by

$$T_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1+\cos^2\theta_1+(k_0-k_1)(1-\cos\theta_1) \\ -\sin^2\theta_1 \\ 0 \end{pmatrix},$$

from Eq. (55). Since these Stokes parameters refer to the $(\mathbf{k}_0, \mathbf{k}_1)$ plane, in order to use Fano's matrix for the second scattering we must
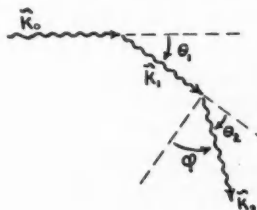

Fig. 5. Double Compton scattering of an unpolarized gamma-ray beam. φ is the angle between the $(\mathbf{k}_0, \mathbf{k}_1)$ plane and the $(\mathbf{k}_1, \mathbf{k}_2)$ plane.

transform by the matrix M these Stokes parameters to refer to the $(\mathbf{k}_1, \mathbf{k}_2)$ plane, which is at an angle Φ to the $(\mathbf{k}_1, \mathbf{k}_2)$ plane. Therefore,

$$d\sigma \sim (1/2)(1\ 0\ 0)T_2$$
$$\times \begin{pmatrix} 1+\cos^2\theta_1+(k_0-k_1)(1-\cos\theta_1) \\ -\sin^2\theta_1(\cos^2\Phi-\sin^2\Phi) \\ 0 \end{pmatrix},$$

which yields $d\sigma \sim \gamma_{01}\gamma_{12} - \gamma_{01}\sin^2\theta_2 - \gamma_{12}\sin^2\theta_1 + 2\sin^2\theta_1\sin^2\theta_2\cos^2\Phi$, where $\gamma_{01} = k_1/k_0 + k_0/k_1$; $\gamma_{12} = k_2/k_1 + k_1/k_2$, which is the same result given by Wightman[12] using the density matrix.

Next let us consider the polarization of electrons as a result of a scattering experiment. If we write the wave function ψ for an electron in terms of orthogonal states of longitudinal spin

$$\psi = a_1\psi_1'' + a_2\psi_2'',$$

then the spin direction of the electron is given by Eq. (29). In terms of these spin angles Mott[13]
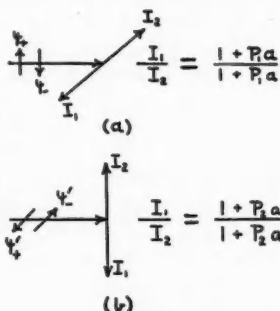
Fig. 6. Single scattering of electrons for the determination of the orientation coefficients with respect to the fundamental spin states shown by the small vectors on the incident beam. $a = D/\bar{I}$.

has shown that the scattered intensity is given by

$$I = \bar{I} - D\sin\theta\sin(\Phi - \pi/2)$$

for right angle scattering, where $\bar{I}$ and D are functions of the scattering angle. Tolhóek[7] has pointed out that the orientation coefficients of the electron beam can then be determined by the two right angle scattering experiments shown in Fig. (6). A measurement of the intensities I_1 and I_2 in each of the two positions shown yields the orientation coefficients P_1 and P_2, where positive values of P_i mean a spin orientation

[12] A. Wightman, Phys. Rev. 74, 1813 (1948).

[13] N. F. Mott, Proc. Roy. Soc. (London) A124, 425 (1929), also N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, New York, 1950), second edition, p. 76.
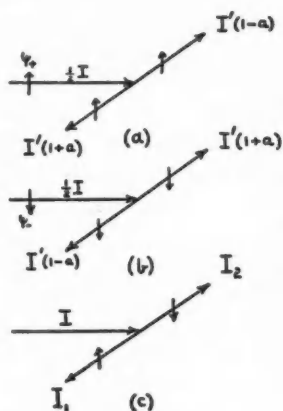
FIG. 7. In (a) and (b) we have an analysis of an unpolarized beam into two independent components of opposite spin. Each component will be scattered with a different intensity so that the resulting beams I_1 and I_2 of the unpolarized beam shown in (c) will be partially polarized. The arrows in the figure show the predominant spin direction of the partially polarized beams.

orthogonal to the plane of scattering as shown in the figure. As an example, for pure states of spin, ψ_+, i.e., $P_1 = 1$, we have

$$\frac{I_1}{I_2} = \frac{\bar{I} - D\,\sin\pi/2\,\sin(-\pi/2)}{\bar{I} - D\,\sin\pi/2\,\sin(\pi - \pi/2)}$$

$$= \frac{\bar{I} + D}{\bar{I} - D} = \frac{1 + a}{1 - a}, \quad (56)$$

where $a = D/\bar{I}$. For a partially polarized beam then the measurement of I_1 and I_2 will yield

$$I_1/I_2 = (1 + Pa)/(1 - Pa),$$

where the P's refer to transverse polarization. Now longitudinal polarization cannot be measured by the scattering experiment alone, just as circular polarization of a beam of light cannot be measured by a Nicol prism alone. The longitudinal polarization must be converted into transverse polarization just as circular polarization must be transformed into plane polarization by means of a quarter-wave plate. For electrons Tolhoek shows that this transformation may be

obtained by the use of a transverse electric field, which changes the direction of motion of the electron, but leaves its spin orientation in space unchanged.

We have seen the analyzing properties of electron scattering experiments from the above discussion; now let us investigate the polarization of an unpolarized electron beam by single scattering. Since scattering is sensitive only to spin states orthogonal to the scattering plane we should expect that the scattered beam would be partially polarized orthogonal to the plane of scattering which is indeed the case. Now an unpolarized beam of electrons can be thought of as an incoherent superposition of two beams of opposite polarization. Hence, an unpolarized beam can be subdivided into the two components shown in Fig. 7(a) and (b). These two components will be scattered differently. Hence, the scattered beams I_1 and I_2 as shown in Fig. 7(c) will be partially polarized with the predominant spin direction as shown in the figure.

CONCLUSION

By the introduction of the Stokes parameters for the description of polarization, once an interaction matrix has been found, the mathematical description of polarization reduces to a very simple form. This method has the advantage that it shows the direct analogy of polarization of particles and high-energy photons to the polarization of light studied in optics courses.

In a letter to Thomas Young in 1824 Fresnel says, "It seems to me, however, that what you left me to do in those parts of optics was as difficult as what you yourself had done. You had gathered the flowers, may I say with English modesty, and I have dug painfully for the roots. . . . All of the compliments which I have been able to receive from MM. Arago, Laplace, or Biot have never given me so much pleasure as the discovery of one theoretical truth and the confirmation of my calculations by experiment."—Thomas Young, Natural Philosopher, by ALEX WOOD (Cambridge University Press, 1954).

Clifford N. Wall

Recipient of the 1953 Oersted Medal for Notable Contributions to the Teaching of Physics

The American Association of Physics Teachers has conferred upon Clifford N. Wall, Professor of Physics at the University of Minnesota, the eighteenth of its annual awards for notable contributions to the teaching of physics. The address of recommendation printed below was made by Mark W. Zemansky for the Committee on Awards, and the presentation of the medal and certificate was made by Paul E. Klopsteg, President of the Association, at a ceremony in McMillin Theater, Columbia University, New York, New York, on January 29, 1954, during the twenty-third annual meeting.
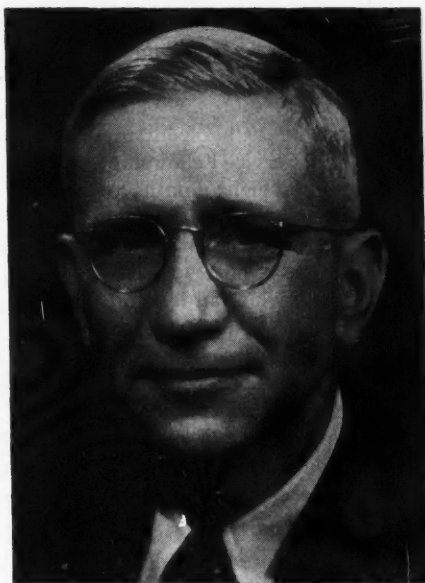
Presentation of the Oersted Award to Professor Clifford N. Wall

MARK W. ZEMANSKY
Acting Chairman of the Committee on Awards for 1953
(Received February 8, 1954)

Remarks made on the occasion of the presentation of the Oersted Award to Professor Clifford N. Wall.

THE rewards of the research physicist are often numerous. The eminent research physicist is promoted rapidly, at times to a specially endowed professorship, and is eligible for medals, prizes, remunerative lecture engagements, and other awards. He is interviewed on television, quoted in the daily papers, and consulted by Government leaders. The eminent teacher, on the other hand, is often treated as though he were partly a work horse and partly a loafer. To make matters worse, at least six times a year he is forced to listen to that nauseating libel which is meant to be so humorous, namely, that "He who can does, but he who can't teaches." The American Association of Physics Teachers, devoted, at least in part, to the task of raising the morale of teachers and of educating the public to understand and to value the work of teachers, is one of the very few organizations that has established a tangible award in recognition of superior contributions to the art of teaching. This award is the Oersted medal, and at this joint meeting of the American Physical Society and the American Association of Physics Teachers we are happy to present our eighteenth medal to another really great teacher, Professor CLIFFORD N. WALL.

Clifford Wall was born in Dayton, Ohio, and attended its public schools. He was attracted to the subjects of physics and mathematics while serving as an assistant in physics at Stivers High School in Dayton. His undergraduate work was spread among three colleges, North Central College (formerly known as Northwestern College), Carnegie Institute of Technology, and the University of Illinois. Continuing at Illinois for graduate work, he received his Ph.D. under Professor Kunz in 1926. While carrying on his

graduate study and research, Dr. Wall acted as half-time teaching assistant under the direction of Robert F. Paton, the present Secretary of our Association.

The year 1928–29 he spent in France on an American Field Service Fellowship—four months in Strasbourg and eight months at the Sorbonne in Paris. Returning to the United States in 1929, Dr. Wall became Professor of Physics at North Central College, a post vacated by Professor Rogers D. Rusk who had left for Mt. Holyoke. During his twelve years at North Central, Dr. Wall stimulated and inspired a surprisingly large number of young men to enter the field of physics. His extraordinary enthusiasm and ability in the difficult job of teaching physics in a small liberal arts college won for him, a number of years later, the Research Corporation Award for distinguished teaching.

A leave of absence from North Central College during 1937–38 enabled him to spend a year at M.I.T. for research on distribution functions for liquids. In 1942 he was called to the University of Minnesota where he now holds the rank of Professor of Physics. Although Professor Wall is primarily a teacher, he has from time to time devoted himself to a considerable amount of research activity. He has always held that a good teacher should do some kind of creative work along with his teaching duties in order to sustain his enthusiasm. He is the author of ten research articles, three articles on the teaching of physics and, in collaboration with R. B. Levine, a Physics Laboratory Manual, published by Prentice-Hall.

I am very pleased to ask PROFESSOR WALL to receive from the hands of President Klopsteg the Oersted award that he so richly deserves.

Metaphysics of a Physics Teacher

C. N. WALL
University of Minnesota, Minneapolis, Minnesota
(Received April 15, 1954)

I WISH to express my deep gratitude for the honor of being awarded the Oersted Medal. In accepting this award, I can do so in good conscience only if I regard the honor as a reflection on the good works of my former physics students. But I have long felt that the greatest honor should go, not to the recipient of the Oersted Medal, but to the few men of imagination who first conceived the idea of an Association of Physics Teachers and an award for good teaching.

As many of you know, teaching, like virtue, is generally regarded as its own reward; and by inference, the greater the teaching load, the greater the reward.

The trouble with the rewards of teaching is that they require some prior responsibilities. I now find that the award of the Oersted Medal also carries with it a responsibility to which I had better devote some attention.

Some of you no doubt wonder about the title of the address which appears after my name in the program. You are not alone. I too wonder about it. In explanation I can only say that it was pulled out of me under duress by Bob Paton and K. K. Darrow long before I had set pencil to paper. Only when I started to do this, did the full nature of my folly become apparent. But the first step in this operation, like the first step in sin, turned out to be irreversible, and I had to hew my way through to the end. In fact, I have spent so much time doing this, and so little time preparing my general physics lectures for the past few weeks, that had the Committee on Awards heard any of those lectures, they would have withdrawn the award forthwith.

For some time I have held the conviction that what a physics teacher thinks, or doesn't think, about certain metaphysical problems concerning the theory of knowledge is bound to

affect his teaching. If he has reflected on these problems at all, he is likely to have in the back of his mind a set of suppositions, more or less explicit, which he can draw on from time to time in order to chart his course in classroom and laboratory. If he has not reflected on these problems, he will probably passively accept a set of presuppositions characteristic of the climate of opinion about him, i.e., he will exhibit a common sense attitude toward these problems. Unfortunately, as Bertrand Russell[1] has pointed out, common sense notions quite generally suffer from three defects: they are cocksure, vague, and self-contradictory. The first step to be taken is to become aware of these defective notions in order to replace them by ideas which are tentative, precise, and self-consistent.

The metaphysical problem of the origin and nature of knowledge with its core question: How do we know?—has enticed and plagued philosophers for centuries. Socrates speculated about it 500 years before Christ; William James was still vitally concerned with the problem 2500 years later. And although the time span of this problem is great, the span in proposed solutions is even greater. These range from Bishop Berkeley's solution which denied the reality of matter, to David Hume's solution which denied the reality of mind. Hence the two of them together managed to eliminate both mind and matter. Whether the phrases "never mind" and "no matter" have any connection with these philosophies, I do not know.

You need have no fear that I am going to launch into a discussion of the various theories of knowledge. What I wish to do, however, is to examine briefly two or three aspects of the problem which seem to have a direct bearing on the teaching of physics.

In matters of this kind one is always safe in starting with Socrates. You may recall that, in Plato's *Meno*, Socrates suggests that "all inquiry and all learning is but recollection" since the soul is immortal and has been born again many times. As proof of this he calls to his side one of Meno's slave boys, and, by means of diagrams in the sand and a series of questions, purports to draw out of this ignorant boy the

[1] Bertrand Russell, *Philosophy* (W. W. Norton & Company New York, 1927), p. 1.

theorem of Pythagoras. Up to a certain point in the argument, the responses of this boy to the questions of Socrates are given without any show of spirit. They run: yes, there are, four, true, no, etc. Suddenly Socrates leads him into a trap and confronts him with a dilemma. The boy says: "Now, by Jove, Socrates, I do not know." This is the first sign of learning on the part of the boy and the first indication that the boy is really taking any part in the discussion. He has virtually been shocked into participation. Socrates remarks to Meno: "Is he not better off in knowing his ignorance? . . . If we have made him doubt, and given him the 'torpedo's shock,' have we done him any harm? . . . We have certainly done something that may assist him in finding out the truth of the matter; and now he will wish to remedy his ignorance, but then he would have been ready to tell all the world that the double space should have a double side."

One can hardly believe that this example proves that all learning is but recollection. Fragments, however, of this particular theory of learning exist even today in many of our best physics laboratories. In his *Essay on Criticism*, Alexander Pope judiciously observed: "Men must be taught as if you taught them not; and things unknown proposed as things forgot."

But this Socratic episode also includes the germs of the valuable educational principle that the process of learning consists not only in what is brought to the learner, but also, in what is drawn out of him. Or, to put it in more appealing terms for physicists, the learning process is at once a process of absorption and a process of emission; the learner, both a passive absorber and an active emitter. Thus there is a certain polarity in the learning process and it is folly for the teacher to neglect either pole. Unfortunately this is precisely what we are inclined to do. In today's progressive schools, I am told, the little learners are primarily little emitters, whereas in the old red school house days they were primarily little absorbers.

I know of no place where it is more appropriate to apply Aristotle's principle of the golden mean than in the educative process. Here the virtue of a teacher consists in taking his stand somewhere between the extreme right position of pure

absorption and the extreme left position of pure emission.

An educational Juggernaut—I use the word advisedly—which exemplifies the extreme right in education is the large lecture section in physics. Here the student becomes a pure absorber. He finds it virtually impossible to get any sense of participation in the events which are occurring way down in the front of the lecture room. Without this sense of participation, he never feels quite at home, but is rather like a traveler in a foreign land. If he is smart enough, he is able to find his way about, perhaps with the aid of a Baedeker. It is virtually impossible to put him in an excited state; he acts for all the world like a perfect absorber with a capacity for infinite zero-point energy. And when the course is over, he doesn't choose to continue in physics.

The lecturer, on the other hand, must necessarily be a pure emitter. His task can be fittingly compared with that of Sisyphus in Hades. Sisyphus, you may remember, was condemned to roll to the top of a hill a huge stone which promptly rolled back down again, thus making his task endless.

I must hasten to admit that this picture of the large lecture section in physics which I have just given is somewhat exaggerated. It need not be quite that bad. I gave the picture simply in order to bring out the inherent tendency of this system to swing to the extreme right, if left to itself. And the endless task of the teacher, if he is to keep the system even a few degrees away from its inferior limit, is indeed Sisyphean in character.

In the citation Professor Zemansky referred to the "difficult job of teaching physics in a small liberal arts college." I trust that no one will infer from this that the corresponding job of teaching undergraduate physics in a large university is easy by comparison. I have tried my hand at both of these jobs and have found the opposite to be more nearly the case. My experience puts me in full accord with Emerson when he says: "I think no virtue goes with size." Of course, he was talking about the titmouse, and I am talking about something else.

The something else is the character of the education, whether its source is in the big university or the small college. Good education is not mere knowledge transferred from teacher to student. If it is, there has been no need for teachers and universities and colleges since the invention of the printing press. It is the way in which this knowledge is transferred that is all important. It must be conveyed with imagination and with zest. The distinguished philosopher, Alfred North Whitehead, puts it this way: "The justification for a university is that it preserves the connection between knowledge and the zest of life by uniting the young and the old in the imaginative consideration of learning. The university imparts information, but it imparts it imaginatively. . . . A University which fails in this respect has no reason for existence."[2]

These are strong words but they are fair ones. They present a distinct challenge to any university or college, large or small. The big university most often fails to meet this challenge because of the *impersonal* character of its education, especially among the undergraduates. Zest and imagination simply do not square with impersonal relations between student and teacher.

On the other hand, the small college often fails to meet this challenge because it allows its faculty members to lose their zest for knowledge by not encouraging research work, or, worse still, because it is unable to get competent faculty members in the first place.

The merit of an institution of higher learning, whether large or small, depends primarily on how well it has solved this problem.

I now wish to turn to another aspect of the theory of knowledge which bears directly on physics and the teaching thereof.

There is a widespread doctrine in the teaching of physics, appropriately re-enforced by most textbooks, that the way to cover any phase of the subject is to go, usually as quickly as possible, from a set of facts or particulars to a few general principles. This is called induction. The particulars or facts may simply be stated as such, or they may be pulled out of demonstration experiments, or they may be obtained in the laboratory. At all events, once the general principles have been reached, the teacher breathes a sigh of relief, and begins assigning

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[2] Alfred North Whitehead, *The Aims of Education* (Mentor Book M 41, The New American Library, New York, 1949), p. 97.

problems. The student is then expected to reverse the process by going from the principles to the particular solution of a problem. This is called deduction.

From the viewpoint of the student, the first process is somewhat like putting a jigsaw together, with many of the pieces missing; the second process involves considerable guess work as to the nature of the missing pieces.

Again I have, no doubt, been guilty of oversimplification. But as I examine my own procedure in teaching physics, or hear what my colleagues are doing in this respect, or read the textbooks, the foregoing is approximately what I observe. And if you ask me how else one should proceed, then I would be rather hard put to give an answer.

I should like to make two observations about this method which I think are pertinent.

The first one is a simple matter of logic which the teacher, especially the one well trained in physics but lacking in teaching experience, is quite likely to overlook. The observation is this: one cannot logically draw out of a proposition concerning universals any information concerning particulars. If there are no particulars in the proposition, then no particulars can be gotten out of it. To give a trite illustration: the proposition that "all men are mortal" taken by itself allows us to draw no conclusion whatever about Socrates. It is not until it is combined with the minor premise, "Socrates is a man," that we are able to draw conclusions about Socrates.

We are prone to get impatient with a student who cannot, for example, solve a simple Atwood problem by applying Newton's laws of motion, and turning the crank. We should not be surprised at this; no one can do it. What happens, of course, is that when we work the problem we not only use Newton's laws of motion but we also draw on a substratum of minor premises laid down by experience, but seldom exposed. The student has no such substratum of knowledge and hence is helpless no matter how well he fancies he knows the laws of motion. In one sense, progress in physics largely consists in building up this reserve supply of minor premises, such things for example as: gold is a metal; weight is a force; velocity is a vector quantity; this cord is inextensible, etc. Without a goodly supply of such particulars the general principles of physics are impotent.

There are good reasons for insisting that a student in physics work problems both on paper and in the laboratory. Not the least of these reasons is that this procedure contributes to the student's reserve supply of particulars. "It is the very essence of a good problem that it offers something new, and must be thought out by itself," as has been so aptly stated by Frederick A. Saunders.[3] But it should be realized by teacher and student alike that the process of working a good physics problem is not simply one of pure deduction. It is just at this point that the student needs individual help; help that cannot be provided by a lecture section.

I have frequently noted that certain textbooks which are distinctly appealing to physics teachers are, at the same time, distinctly appalling to the student, and *vice versa*. I have a feeling that part of this difference in attitude rests directly on the point under discussion. The teacher shudders to see a physics book cluttered up with a lot of details and particulars which appear to hide the beauty and meaning of the principles. Why all this claptrap, he snorts; it is impossible to see the forest for the trees. The student, on the other hand, is in a very different position. He hardly knows a tree when he sees one and he surely cannot distinguish between a birch and an elm. He therefore appreciates the so-called claptrap because it seems to be the only way for him to make progress.

The other observation which I wish to make about the traditional procedure in teaching physics has not to do with logic but rather with the character of the course of mental development of the beginning physics student. This course of mental development need not, and frequently does not, correspond with the course of the development of the subject matter in physics. If it did, one would suppose that the physics student would first have a clear conception of individual facts which would gradually evolve into a clear conception of fundamental principles. But experience seems to indicate that the actual growth of knowledge in an individual is not one from clear ideas of particulars to clear

[3] F. A. Saunders, *A Survey of Physics* (Henry Holt and Company, New York, 1943), third edition, p. 73.

ideas of universals but rather from *vague* to *definite* ideas, whether the ideas concern particulars or universals.

Morris Cohen in his book on *Reason and Nature* puts the matter this way: "That all knowledge begins with perception of the particular and then goes on by abstraction to the universal is a widespread dogma. It probably arises from the fact that a good deal of our education consists in being taught to name and recognize certain abstract phases of existence, and as we cannot suppose that children before they learn to talk have such general ideas, we conclude that they can come only after the perception of particular things. But careful attention to the actual growth of knowledge shows that it is mainly a progress not from particular to universal but from the vague to the definite. . . . In ordinary life we perceive trees before we perceive birches. . . . It is therefore quite in harmony with fact to urge that perception of universals is as primary as the perception of particulars. The process of reflection is necessary to make the universal clear and distinct, but as the discriminating element in observation it also aids us to recognize the individual."[4]

If this observation is correct, one is forced to conclude that the beginner in physics starts out with a lot of very fuzzy notions and experiences. He progresses by converting these fuzzy items into more or less definite ideas and experiences. There appears to be no immutable order in which this progression occurs. It seldom

follows the logical order in which the material is presented—if the material is so presented. Later material often serves to clear up earlier material. It is just as important for a teacher to know when to stop talking about a certain subject as it is to know when to introduce the subject.

Finally, there is no guarantee whatever that a student who "knows" Ohm's law or even Maxwell's equations will therefore know how to connect up an ammeter in the laboratory. Such an assumption is not only false, it is disastrous. I know whereof I speak in this matter, for at Minnesota we give practical laboratory tests as part of the course work. I have seen second-year engineering students who know all about Ohm's law create a shambles of electrical equipment, if given any power source greater than a dry cell. Apparently they have not yet reached the point where they are able to distinguish between one ampere and one hundred amperes.

With this example I bring to a close the metaphysical reflections of a physics teacher. I make no proposals of action; I offer no rules of conduct for the classroom or laboratory. All that I have tried to do is to bring out into the open some of the intrinsic characteristics of the problem of teaching and the problem of learning.

Having opened the discussion on a Socratic note, I will close it in a similar manner by paraphrasing one of his comments: I do not mean to affirm that what I have been saying is exactly true—a man of sense ought hardly to do that. But I do say that something of the kind is true.

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[4] Morris Cohen, *Reason and Nature* (Harcourt Brace and Company, New York, 1931), p. 124.

Erratum: A Note on the Study of Uniformly Accelerated Motion

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On page 254 of the above paper, lines 11 and 10 from the end should read: "get the acceleration as having the value 1.428 ± 0.009 cm/interval/interval, which is to be etc."

Some Elementary Concepts in Electrostatics

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(Received October 10, 1953)

This note points out that Gauss's law is insufficient, except in special cases, to solve simple problems in electrostatics. In any system of dielectrics and charged conductors an additional condition, *viz.*, that the total free energy is at a minimum is necessary to determine the equilibrium values of the fields. As will be seen, this requirement amounts to the statement that in electrostatics the potential across any conductor is continuous. For the sake of generality, the permittivities of the dielectric media are considered to be inhomogeneous, temperature dependent and also field strength dependent. The last property being included in case some (or all) of the dielectrics have the perovskite structure as, for example, in the titanates, tantalates, niobates, zirconates, etc. Consideration is given to interesting special cases of the generalized results.

I. CHARGED PLANE CONDUCTOR

CONSIDER, as a first example, a plane conductor of area A per side and bearing a total charge Q sandwiched between two media characterized by dielectric constants $\mathcal{E}_1(x_1)$ and $\mathcal{E}_2(x_2)$ (see Fig. 1). Gauss's law applied around the dotted surface shown produces the equation

$$D_1 + D_2 = 8\pi\bar{\sigma}, \qquad (1)$$

where $\bar{\sigma} = Q/2A$ is the *average* superficial charge density on the conductor. Applying Gauss's law about any other similar contour wholly inside either of the dielectric media yields the result that D_1 and D_2 are independent of their respective coordinates. In order to find these displacements uniquely, a second equation independent from Eq. (1) is required. It is quickly seen that such a result is *not* obtained by applying Gauss's law about other surfaces which cut through the conductor for it is not known what fraction of the total charge Q is on either of its surfaces. In addition, since the field in the conductor is zero, there is no continuity of the displacement vector across it. To solve the problem it is clear, since the whole system must be in thermodynamic equilibrium, that the total free energy must be at a minimum.

The steps necessary to calculate the free energy of a non-lossy temperature dependent dielectric are given in Fröhlich,[1] and the extension to the case where the permittivity depends on field strength as well as temperature is straight-

forward. For a dielectric characterized by a permittivity $\mathcal{E}(T,E)$ such that $\mathcal{E} \to \mathcal{E}_0(T)$ as $E \to 0$ the displacement vector may be written

$$D = D(\mathcal{E}_0(T), E). \qquad (2)$$

It is then easy to show that the free energy per unit volume of dielectric is

$$F = F_0(T) + \frac{1}{4\pi}\int_D E(D, \mathcal{E}_0)dD, \qquad (3)$$

where $F_0(T)$ is the free energy per unit volume in zero field strength at a point where the temperature is T. Applying Eq. (3) to the components of Fig. 1, the total free energy is

$$\mathfrak{F} = \mathfrak{F}_0 + \frac{A}{4\pi}\int_{x_1=0}^{\infty}\left\{\int_{D_1} E_1 dD_1\right\}dx_1$$

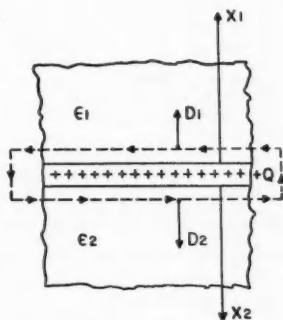$$+ \frac{A}{4\pi}\int_{x_2=0}^{\infty}\left\{\int_{D_2} E_2 dD_2\right\}dx_2,$$

FIG. 1. Gauss's law applied to a plane conductor.

[1] H. Fröhlich, *Theory of Dielectrics* (Oxford University Press, London, 1949), pp. 9–12.

where $\mathfrak{F}_0$ is the contribution from F_0 in the dielectrics and the conductor. Varying D_1 to obtaining a minimum in $\mathfrak{F}$

$$\frac{\delta\mathfrak{F}}{\delta D_1}=\frac{A}{4\pi}\int_{x_1=0}^{\infty}E_1 dx_1+\frac{A}{4\pi}\int_{x_2=0}^{\infty}E_x\frac{\delta D_2}{\delta D_1}dx_2=0$$

and using Eq. (1) results in the equation

$$\int_{x_1=0}^{\infty}E_1(D_1,\ _1\mathcal{E}_0)dx_1$$
$$-\int_{x_2=0}^{\infty}E_2(D_2,\ _2\mathcal{E}_0)dx_2=0, \quad (4)$$

which states that the potential difference between $-\infty$ and $+\infty$ is zero. Equations (1) and (4) are sufficient to determine the fields. Two subcases are possible.

(a) Nonfield Dependent Permittivities

Suppose that $D=\mathcal{E}_0 E$ in the above dielectrics, then Eqs. (1) and (4) may be written

$$\left.\begin{array}{r}D_1+D_2=8\pi\bar{\sigma}\\\alpha_1 D_1-\alpha_2 D_2=0\end{array}\right\}, \quad (5)$$

where

$$\alpha_1=\int_{0}^{\infty}\frac{dx_1}{_1\mathcal{E}_0(x_1)},\quad \alpha_2=\int_{0}^{\infty}\frac{dx_2}{_2\mathcal{E}_0(x_2)}.$$

The set of Eqs. (5) have a solution

$$\left.\begin{array}{l}D_1=E_1(x_1)\ _1\mathcal{E}_0(x_1)=8\pi\bar{\sigma}\left(\dfrac{\alpha_2}{\alpha_1+\alpha_2}\right)\\[2mm]D_2=E_2(x_2)\ _2\mathcal{E}_0(x_2)=8\pi\bar{\sigma}\left(\dfrac{\alpha_1}{\alpha_1+\alpha_2}\right)\end{array}\right\}, \quad (6)$$

which, for the case of homogeneous dielectrics, become

$$E_1=\frac{8\pi\bar{\sigma}}{_1\mathcal{E}_0+_2\mathcal{E}_0}=E_2. \quad (7)$$

If α_1 diverges (as in an homogeneous dielectric) while α_2 converges (as in a dielectric whose permittivity increases with increasing distance from the conductor) then from Eq. (6)

$$\left.\begin{array}{l}D_1=0\\D_2=8\pi\bar{\sigma}\end{array}\right\}. \quad (8)$$

The result Eq. (8) may have a certain application to the problem of the aurora in which it is thought that sheets of charged particles penetrate the earth's atmosphere from outer space. Consider an isothermal atmosphere at a temperature T, then the density at any point z above the earth's surface is

$$\rho=\rho_0 e^{-z/H},$$

where ρ_0 is the density at ground level and $H=kT/mg$, m being the mean molecular mass. The variation of permittivity with height is then

$$\mathcal{E}=1+4\pi\rho_0(\alpha/m)e^{-z/H},$$

where α is the average molecular polarizability. Thus in terms of the coordinates used in Fig. 1 the dielectric constants are

$$_1\mathcal{E}_0=1+\beta(h)e^{-x_1/H}$$
$$_2\mathcal{E}_0=1+\beta(h)e^{x_2/H},$$

where the region above the sheet is identified with medium 1 and that below with medium 2. In these equations h is the height of the sheet above the earth's surface and $\beta(h)=4\pi\rho_0(\alpha/m)\times e^{-h/H}$. It is quickly seen that

$$\alpha_1=\int_0^{\infty}\frac{dx_1}{1+\beta(h)e^{-x_1/H}}\to\infty,$$

$$\alpha_2=\int_0^{\infty}\frac{dx_2}{1+\beta(h)e^{x_2/H}}=H\log\left(\frac{1+\beta}{\beta}\right),$$

and it follows that the fields above and below the sheet are

$$E_1=0,$$
$$E_2=\frac{8\pi\bar{\sigma}}{1+4\pi\rho_0(\alpha/m)e^{-(h-x_2)/H}}. \quad (9)$$

Assuming the air to be non-lossy, neglecting the presence of a dielectric and conducting earth, and overlooking the transient nature of the phenomenon may, of course, be sufficient reason that Eq. (9) would not hold in the practical case.

(b) Field Dependent Permittivities

When the dielectric constant depends on field strength, Eq. (4) must be integrated knowing

the function

$$E = E(D, \mathcal{E}_0(x)).$$

If the dielectric media are in zero temperature gradient and otherwise homogeneous the relations (1) and (4) become

$$\left. \begin{array}{l} D_1 + D_2 = 8\pi\bar{\sigma} \\ E_1 - E_2 = 0 \end{array} \right\}, \qquad (10)$$

and for an explicit solution the dependence of the displacements on field strength must be known. In certain solid solutions of the titanates, for example, Roberts[2] has shown that above the Curie temperature

$$E = D/\mathcal{E}_0 + \eta D^3, \qquad (11)$$

where $\mathcal{E}_0(T(x),x)$ is the dielectric constant in "weak" fields and $\eta(x)$ is a constant independent of temperature and field strength.

II. PARALLEL PLATE CONDENSER

As a second example, consider the case of two infinite plane conductors bearing charges $+Q$ and $-Q$, respectively, (Fig. 2).

The only two independent results obtainable by applying Gauss's law are

$$D_1 + D = 8\pi\bar{\sigma}, \qquad (12)$$

$$D + D_2 = 8\pi\bar{\sigma}, \qquad (13)$$

where as before $\bar{\sigma} = Q/2A$ is the average surface charge density for each conductor. Applying Gauss's law about other similar surfaces wholly in one or other of the dielectric media shows that the displacements are independent of position. A third equation is supplied by minimizing the total free energy with respect to any one of the displacements

$$\mathfrak{F} = \mathfrak{F}_0 + \frac{A}{4\pi}\left\{ \int_{x=0}^{d}\left[\int_D EdD\right]dx \right.$$

$$\left. + \int_{x_2=0}^{\infty}\left[\int_{D_2} E_2 dD_2\right]dx_2 + \int_{x_1=0}^{\infty}\left[\int_{D_1} E_1 dD_1\right]dx_1 \right.$$

leading to

$$\int_0^d Edx - \int_0^\infty E_2 dx_2 - \int_0^\infty E_1 dx_1 = 0. \qquad (14)$$

[2] S. Roberts, Phys. Rev. **71**, 890 (1947).

As before, the dependence of E on D must be known for an explicit solution.

(a) Nonfield Dependent Permittivities

When $D = \mathcal{E}_0 E$, Eqs. (12), (13), (14) become

$$\left. \begin{array}{l} D + D_1 = 8\pi\bar{\sigma} \\ D + D_2 = 8\pi\bar{\sigma} \\ \alpha D - \alpha_1 D_1 - \alpha_2 D_2 = 0 \end{array} \right\}, \qquad (15)$$

where

$$\alpha = \int_0^d [dx/\mathcal{E}_0(x)], \quad \alpha_1 = \int_0^\infty [dx_1/{_1}\mathcal{E}_0(x_1)],$$

$$\alpha_2 = \int_0^\infty [dx_2/{_2}\mathcal{E}_0(x_2)].$$

The solutions are thus

$$\left. \begin{array}{l} D_1 = D_2 = 8\pi\bar{\sigma}(\alpha/\alpha + \alpha_1 + \alpha_2) \\ D = 8\pi\bar{\sigma}(\alpha_1 + \alpha_2/\alpha + \alpha_1 + \alpha_2) \end{array} \right\}. \qquad (16)$$

If α_1 and α_2 diverge, as in homogeneous media, these fields become

$$\left. \begin{array}{l} E_1 = E_2 = 0 \\ D = \mathcal{E}_0(x)E(x) = 8\pi\bar{\sigma} \end{array} \right\}. \qquad (17)$$

On the other hand if α_1 and α_2 converge (as in media for which the dielectric constants increase with increasing distance from the plates), the fields outside the condenser are not zero and must be determined from Eq. (16).

(b) Field-Dependent Permittivities

As before, consider the case that the media are homogeneous, then the fields are constant in
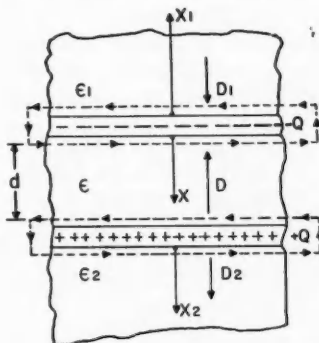

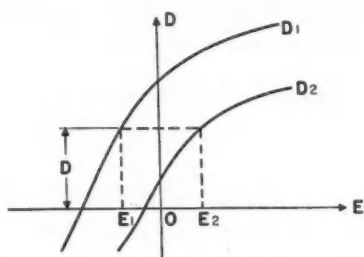FIG. 2. Gauss's law applied to two infinite plane conductors.

FIG. 3. Displacement vectors D_1 and D_2 as functions of the field strength.

Eq. (14) and it follows that

$$D+D_1=8\pi\bar{\sigma}\atop D+D_2=8\pi\bar{\sigma}\atop E_1+E_2=0\Bigg\}. \qquad (18)$$

To solve these equations simultaneously, a knowledge of $D(E)$, $D_1(E_1)$, and $D_2(E_2)$ are required. There are two subcases possible.

(i) if D_1 and D_2 are functions of the field strength which show a remanence (as is true of the perovskites below their Curie temperature), then (from Fig. 3) fields $E_2=-E_1$ can exist which are not zero and the field inside the condenser is determined from Eq. (18).

(ii) If D_1 and D_2 do not show a remanence (as is true of the perovskites above their Curie point), then from Fig. 3 D_1 and D_2 must both pass through the origin. Hence, $D_1=D_2=0$; $E_1=E_2=0$ and $D=8\pi\bar{\sigma}$.

III. THE SPHERICAL CONDENSER

As a final example consider the spherical condenser (Fig. 4). Gauss's law applied around any contour such as S which is wholly in one of the dielectric media yields the result that $D=A/r^2$. So that in the three media shown

$$D_1=A_1/r^2 \quad (r<a)\atop D=A/r^2 \quad (a<r<b)\atop D_2=A_2/r^2 \quad (r>b)\Bigg\}. \qquad (19)$$

Applying this law again about contours S_2 and S_1 cutting the conducting spheres leads to

$$A_2+A=Q, \qquad (20)$$

$$A+A_1=Q. \qquad (21)$$

Finally the total free energy

$$\mathfrak{F}=\mathfrak{F}_0+\int_{r=0}^{a}\left\{\int_{D_1}E_1dD_1\right\}r^2dr$$
$$+\int_{r=a}^{b}\left\{\int_{D}EdD\right\}r^2dr+\int_{r=b}^{\infty}\left\{\int_{D_2}E_2dD_2\right\}r^2dr$$

must be minimized with respect to one of the parameters A_1, A_2, or A. Choosing, say, A, and using the results

$$\frac{\delta}{\delta A}=\begin{cases}\dfrac{\delta}{\delta D}\cdot\dfrac{\delta D}{\delta A}=\dfrac{1}{r^2}\dfrac{\delta}{\delta D} & (a<r<b)\\[2mm]\dfrac{\delta}{\delta D_1}\cdot\dfrac{\delta D_1}{\delta A}=\dfrac{1}{r^2}\dfrac{\delta A_1}{\delta A}\dfrac{\delta}{\delta D_1} & (r<a)\\[2mm]\dfrac{\delta}{\delta D_2}\cdot\dfrac{\delta D_2}{\delta A}=\dfrac{1}{r^2}\dfrac{\delta A_1}{\delta A}\dfrac{\delta}{\delta D_2}, & (r>b)\end{cases}$$

a third equation independent from Eqs. (20) and (21) is obtained; viz,

$$\frac{\delta A_1}{\delta A}\int_0^a E_1dr+\int_a^b Edr+\frac{\delta A_2}{\delta A}\int_b^\infty E_2dr=0.$$

Utilizing Eqs. (20) and (21) this last result becomes

$$\int_a^b Edr-\int_0^a E_1dr-\int_b^\infty E_2dr=0. \qquad (22)$$

(a) Nonfield Dependent Permittivities

If $D=\mathcal{E}_0(r)E(r)$, Eqs. (20), (21), and (22) become

$$A+A_2=Q\atop A+A_1=Q\atop \alpha A-\alpha_1A_1-\alpha_2A_2=0\Bigg\}, \qquad (23)$$
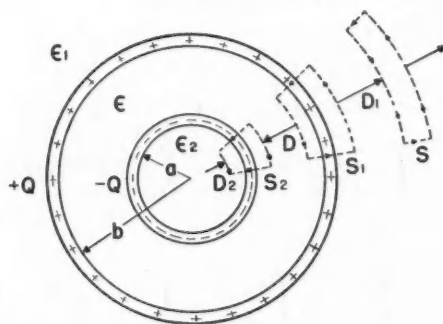
FIG. 4. Gauss's law applied to a spherical condenser.

where

$$\alpha = \int_a^b \frac{dr}{\mathcal{E}_0(r)r^2}, \quad \alpha_1 = \int_0^a \frac{dr}{{}_1\mathcal{E}_0(r)r^2}, \quad \alpha_2 = \int_b^\infty \frac{dr}{{}_2\mathcal{E}_0(r)r^2}$$

and solution is

$$\left. \begin{array}{l} A_1 = A_2 = Q(\alpha/\alpha + \alpha_1 + \alpha_2) \\ A = Q(\alpha_1 + \alpha_2/\alpha + \alpha_1 + \alpha_2) \end{array} \right\}. \qquad (24)$$

Since α and α_2 converge while α_1 diverges, in the homogeneous case

$$A_1 = A_2 = 0$$
$$A = Q.$$

On the other hand, if $\mathcal{E}_1 \alpha 1/r^n$ with $n \geq 2$, the integral α_1 would converge, and the fields inside the small sphere and outside the large sphere would not vanish.

(b) Field Dependent Permittivities

As in the previous examples, consider the case of an homogeneous but field dependent dielectric constant. Then from Eq. (11) for solid solutions of the titanates above their Curie temperature.

$$\int E dr = \int (D/\mathcal{E}_0 + \eta D^3) dr$$

$$= \frac{A}{\mathcal{E}_0} \int (dr/r^2) + \eta A^3 \int (dr/r^6),$$

and Eq. (22) becomes

$$\int_a^b E dr - \lim_{\Delta \to 0} \left\{ \frac{A_1}{{}_1\mathcal{E}_0} \int_\Delta^a \frac{dr}{r^2} + \eta_1 A_1^3 \int_\Delta^a \frac{dr}{r^6} \right\}$$

$$- \int_b^\infty E_2 dr = 0,$$

where it is clear that the integrals $\int_a^b E dr$ and $\int_b^\infty E_2 dr$ converge. Since

$$\lim_{\Delta \to 0} \left| \frac{\int_\Delta^a dr/r^2}{\int_\Delta^a dr/r^6} \right| = 0,$$

it follows that the three equations determining the fields are

$$A + A_2 = Q,$$
$$A + A_1 = Q,$$
$$\eta_1 A_1^3 = 0.$$

Consequently, $A_1 = A_2 = 0$ and $A = Q$. The fields thus become

$$E_1 = E_2 = 0 \qquad (25)$$
$$E = Q/\mathcal{E}_0 r^2 + \eta Q^3/r^6$$

corresponding to subcase (ii) of the parallel plate condenser problem.

Albert E. Caswell 1884-1954

Albert E. Caswell, Professor Emeritus and former Head of the Department of Physics at the University of Oregon, died on June 18th in Eugene, Oregon, at the age of 70. He was born in Winnipeg, Manitoba, and received his doctor's degree from Stanford University in 1911. After two years at Purdue University he came to the University of Oregon in 1913. He was appointed chairman of the department in 1934 and served in that capacity until 1949. He was a National Research Fellow in 1919–20 and during World War II was a member of the staff of the Radiation Laboratory at MIT. Professor Caswell was the author of several papers and of a widely used textbook in general physics.

A Note on the Design of the Spherometer

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(Received January 29, 1954)

A discussion is made of the errors involved in the use of a spherometer, and it is shown that the accuracy is limited by the accuracy with which the radius of the base circle is measured. A suggested change in design is given, which promises to mean easier manufacture and higher accuracy.

IT was the challenging task of the writer years ago to equip the physics laboratory in a Chinese university under the most adverse conditions. War with Japan was going on; it was impossible to place orders for apparatus from this country. We had to make as many different kinds of instruments as we could, and to save labor and cost, we had to deviate from its usual design. Such was the case with the spherometer. Recently, in our department, conversation turned to this instrument, and Dr. M. R. Wehr, our department head, was very much interested. It is at his suggestion that this short note is written.

Let us first go into the question of errors involved. They can be grouped under two types, (1) error due to faulty manufacture, and (2) error inherent in the readings. Under the ideal conditions, we measure the distances a and d, (see Fig. 1) from which the radius r is calculated by means of the equation

$$r = \frac{a^2}{2d} + \frac{d}{2}. \qquad (1)$$

This assumes that the micrometer screw coincides with the axis OA of the base circle of radius a. Errors are introduced if the above condition is not satisfied. One can think of two possibilities: (a) the micrometer screw may make a small angle θ with the axis OA, or (b) it may be parallel to OA, but at a small distance p away from it. The percentage error in d due to (a) is proportional to $1 - \cos\theta$, and ordinarily may be neglected. The error due to (b), which we shall designate as ϵ, may be calculated as follows:

Since

$$d = r - (r^2 - a^2)^{\frac{1}{2}}, \qquad (2)$$

and

$$d' = (r^2 - p^2)^{\frac{1}{2}} - (r^2 - a^2)^{\frac{1}{2}}, \qquad (3)$$

$$\epsilon = d - d' = r - (r^2 - p^2)^{\frac{1}{2}} = p^2/2r. \qquad (4)$$

Using the approximate relation $r = a^2/2d$ from Eq. (1), and substituting into Eq. (4), we get the fractional error in d, given by

$$\epsilon/d = p^2/a^2. \qquad (5)$$

In ordinary spherometers, a is of the order 2.5 cm. If the micrometer screw is off center by half a millimeter, which is hardly probable, we have

$$\frac{\epsilon}{d} = \left(\frac{0.05}{2.50}\right)^2 = \frac{1}{2500}. \qquad (6)$$

This means that the error introduced in d is only 0.04 percent, very much smaller than the error in taking readings. Thus it appears that the disposition of the micrometer screw with respect to the base circle of radius a does not play an important part in the accuracy of the spherometer readings.

Now let us look into the errors inherent in the readings. If we neglect the term $d/2$ in Eq. (1), we have
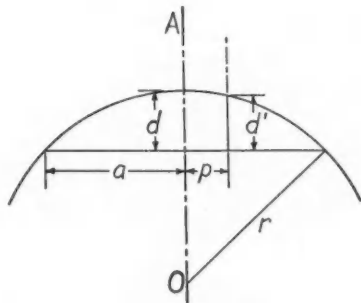
$$r = a^2/2d. \qquad (7)$$

Fig. 1. Geometry of the spherometer.

Differentiating, we get

$$\left|\frac{\Delta r}{r}\right| = 2\left|\frac{\Delta a}{a}\right| + \left|\frac{\Delta d}{d}\right|. \qquad (8)$$

Thus the fractional error in r is the sum of two terms. If one term is very much greater than the other, it means wasted accuracy, because the fractional error in r is nearly equal to the fractional error in the less accurately measured quantity. The best condition is to have the two terms nearly equal. It means that the fractional error allowable in a should be half as large as the fractional error allowable in d. In ordinary spherometers, d can be read to 0.01 mm, and for a value of d about 2 mm, $|\Delta d/d| = 1/200$. This requires $|\Delta a/a| = 1/400$. For an average value of 2.5 cm for a, it means that a should be read accurate to 0.06 mm. This is impossible in the present design, since a is calculated from the measured distance between the tips of the three legs. For values of d greater than 2 mm, its fractional error gets smaller, and the error in r is largely due to that in measuring a. Therefore, there seems to be a genuine need to improve the accuracy with which a is measured.

In view of the above discussion, it was decided to replace the three legs of the spherometer by a hollow cylinder. The inside radius is to be used for convex surfaces, and the outside radius for concave surfaces. Here the radius a may be measured by means of vernier calipers or micrometer calipers. The upper end of the cylinder is threaded and takes the block in which the micrometer screw moves. Thus each micrometer screw block may be fitted with two or more cylinders of different radii. It is hoped that, besides being more accurate, this new type of spherometer may also be easier to manufacture.

An Experiment on the Dielectric Constant of a Gas

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(Received July 29, 1953)

An experiment is described wherein the dielectric constant of air is measured in a student laboratory. By using a beat frequency method, rather than a direct comparison, precisions of the order of 0.01 percent to 0.001 percent are obtainable with ordinary equipment. Important instructional features of the experiment are mentioned.

THERE is probably general agreement that simplicity is a desirable characteristic of an experiment which is to be performed in an instructional laboratory. It is also a good idea, on the other hand, to give the student a chance occasionally to work on a more complex experiment. In this way, he can get a taste of "the real thing." Perhaps this is a way of rationalizing the instructor's desire to have a complex and impressive toy in the laboratory. Be that as it may, the experiment which will be described below is one of the more complicated type which students have an opportunity to perform near the end of a course in electrical measurements in the early part of which they have performed more or less basic experiments.

This experiment involves the determination of the dielectric constant of air, this being the most convenient gas with which to work. This determination not only illustrates the general method of measuring dielectric constants but presents to the student, in addition to a number of incidental features worthy of his notice, an example of an interesting and powerful method of obtaining precision by an indirect measurement of a quantity which would be difficult to measure with precision directly. The student who has had some experience with experimentation can appreciate this feature of the experiment.

Let us look at the general problem. The direct way of measuring the dielectric constant of a substance is to measure the capacitance of a suitably constructed condenser with and without the substance between its plates. The ratio of the first capacitance to the second is the dielectric constant. Unfortunately, the dielectric
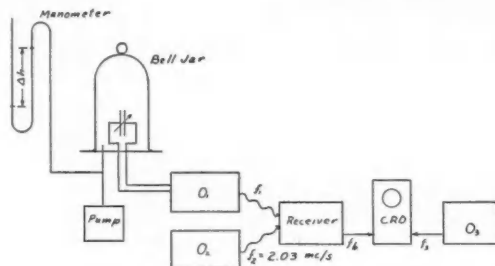
FIG. 1. Block diagram of experimental arrangement for measuring dielectric constant of air.

constant of air is 1.00059. The *direct* measurement of this quantity would need, then, to be made to a precision of 0.01 percent to get even a crude idea of the dielectric constant of air as compared with the dielectric constants of other gases. This difficulty can be surmounted by measuring directly the *change* in capacitance, or in some quantity related to the capacitance, caused by the introduction of the gas into the previously evacuated space occupied by the condenser. In the experiment being described, the condenser in question is in the tank circuit of a radio-frequency oscillator, and we measure the change in oscillator frequency upon admission of the gas to the bell jar. Detailed discussions of these methods are to be found in the literature.[1]

The theory of this method is fairly simple. The frequency of an oscillator containing an $L-C$ oscillatory circuit is given by the equation

$$f = 1/2\pi\sqrt{(LC)}. \tag{1}$$

Strictly speaking, resistance will change the resonant frequency, but in a high-Q circuit, such as we use in our oscillator, this effect is negligible. Differentiating Eq. (1), we get

$$df/f = (-1/2)(dC/C). \tag{2}$$

The capacitance of the condenser can be written as

$$C = kC_0, \tag{3}$$

where k is the dielectric constant of the medium in between the plates of the condenser, and C_0 is the capacitance of the same condenser *in vacuo*.

[1] A good discussion and bibliography will be found in a recent article by J. G. Jelatis in J. Appl. Phys. **19**, 419 (1948).

Differentiating Eq. (3), we get

$$dC/C = dk/k. \tag{4}$$

Therefore,

$$df/f = (1/2)(dC/C)$$
$$= (1/2)(dk/k) \text{ (ignoring sign).} \tag{5}$$

If we let Δk be the difference between the dielectric constant of the gas and that of free space, we have $\Delta k = k - 1$. Therefore,

$$2\Delta f/f \doteq (k-1)/1, \tag{6}$$

since $k \doteq 1$. Solving for k, we get

$$k \doteq 1 + 2\Delta f/f. \tag{7}$$

The problem, then, reduces to that of measuring the change in frequency of an oscillator when the dielectric of the condenser in the tank circuit is changed from vacuum to the gas which is of interest.

In Fig. 1 is shown a block diagram of the apparatus used in this experiment. The test condenser is put under a bell jar which can be evacuated by a mechanical pump. The pressure in the chamber is measured by the difference in height Δh between the two mercury columns in the U-tube manometer. In Fig. 1, O_1 is the oscillator which incorporates the condenser; in our case it operates at a frequency of about 2 mc/sec. Since the frequency change which we wish to measure may be of the order of 500 c/sec, good radio-frequency equipment would be required for its precise determination. However, if we beat the output of this oscillator with that of a constant-frequency oscillator, O_2 in this case, so as to get an audio-frequency signal for the beat note, our problem is then much simplified. The usual way to obtain this beat note is to mix the two outputs directly in some standard mixing circuit. In our case the mixing is done in a receiver which picks up the two radio-frequency signals from O_1 and O_2 directly. (This method reduces the likelihood of interaction between the oscillators.) The audio beat note coming from the receiver is then put into an oscilloscope to which is also connected a calibrated audio oscillator O_3. The beat frequency f_b can thus be measured by comparison with the audio oscillator O_3 by adjusting O_3 till the 1:1

Lissajous figure appears on the screen of the oscilloscope.

We do not follow exactly the procedure suggested by the above discussion, namely to make one measurement of f_b for vacuum and then one for air; but, rather, we determine frequency as a function of air pressure as the air is let into the chamber. If our measurements are plotted on a graph of f_b vs Δh, we get a curve which will show us whether k is a linear function of pressure, and also tell us about the long-term and short-term stability of the oscillator O_1. (A long-term drift of appreciable rate will be apparent from the general shape of the curve, and short-term jumps in the frequency of O_1 will show as obvious discontinuities in the curve.) If the data do not cover a pressure range of one atmosphere, one can compute from the curve the value of Δf_b corresponding to such a change, and get k.

There are two ways in which to take the measurements just discussed; one can make either the pressure or the frequency the controlling factor. In the first method, a small amount of air is admitted into the bell jar, until the pressure is at the desired level, and the beat frequency is measured. This procedure is repeated a number of times, f_b being measured for each pressure. The second method is to

FIG. 3. Further student results. Upper curves show effect of jumps in frequency of O_1 or O_2 and are for wet air. Lowest curve is for dry air.

establish a slow leak into the chamber, set the oscillator O_3 in turn to a number of preselected frequencies, then read pressure as the frequency f_b passes through the frequency set on O_3. The students are inclined to be skeptical of the second method before they do it, because the readings are not stationary, but afterwards they generally decide that their results were better on that run.

Figure 2 shows a plot of typical results. These results are taken from student data, and show what can be obtained with reasonable care. Note the difference between them. The lower curve was taken by the preset pressure method, the upper by the leak method. The points are more consistent in the second set. The explanation presumably is that the second method generally takes less time to perform, so that there is less chance for trouble from oscillator instability Note also that the data points fall on a straight line, showing that $k-1$ is a linear function of pressure. This is to be expected, since $k-1=\chi$, the susceptibility.

Figure 3 illustrates the effect of oscillator instability. In this case the two upper curves bear the same relationship to one another as in Fig. 2, and the data seem to be of about equal merit. But note that in each case there is a group of points displaced from the main set, but lying
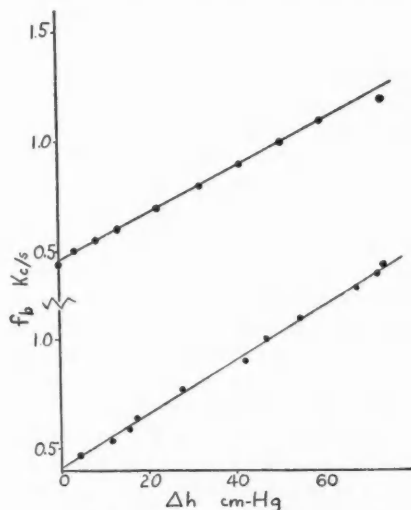

FIG. 2. Typical student results. Upper curve is for continuous leak method, lower curve for preset pressure method.

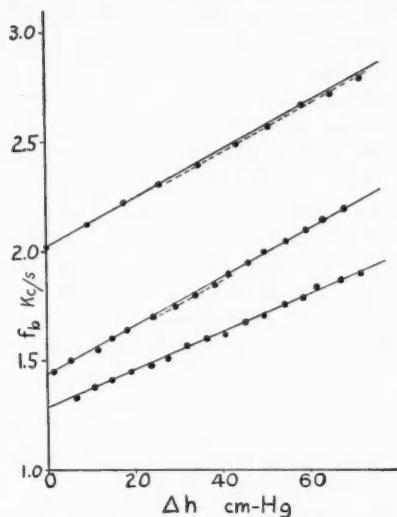on a line of the same slope. This effect results from sudden small shifts in oscillator frequency, which are noticeable if you are listening to the beat note at the time. The students are asked to watch for this effect, since ignoring it could lead them to draw a curve of the wrong slope.

The third curve in Fig. 3 shows the results obtained when the incoming air is dried, in distinction to the others which were taken by admitting air directly from the room. The difference in slope in the two cases is noticeable. The results for $k-1$ obtained from these curves are 0.00085 and 0.00082 for wet air and 0.00062 for dry air, at room temperature. For comparison with other measurements, these should, of course, be reduced to 0°C. The effect of water vapor seems quite large. When one considers, however, that the dielectric constant of water is 81 it becomes reasonable to suppose that the amounts of water present as vapor could cause large effects. As a matter of fact, the students are occasionally asked to see if they can calculate the effect of the water vapor from available data.

In this discussion the effect of stray capacitance has been ignored. The effect of stray capacitance can be appreciable, as can be seen from a study of the oscillator circuit used in this measurement.[1] Whether or not the effect is large enough to worry about depends on what you wish to accomplish. If you should wish to measure the susceptibility, $\chi = k-1$, directly to a precision of better than 1 percent the stray capacitance cannot be ignored. In this student laboratory, however, considerably greater error can be tolerated. Suppose, for instance, that we should find $k-1$ to be 0.00060±10 percent. This is an unimpressive result for a measurement of susceptibility, but when the error in k is computed, it turns out to be 0.006 percent, an error below what you could reasonably expect to achieve by a direct method of measuring ratios. Since this possibility of achieving good accuracy by an indirect method of measurement was one thing to be demonstrated by the experiment, the power of this method has been well demonstrated. This method of measuring directly the difference between two quantities I have called,

for lack of another name, the *method of differences*. It is in principle the method which is used in the Carey-Foster bridge, where two nearly equal resistors are compared by measuring their difference.

This experiment does not require elaborate or expensive equipment; most, or all of the items needed are available in the average physics department. Reasonably good results can be obtained without elaborate precautions. For instance, we have found that an ordinary regenerative oscillator using tickler coil feedback is quite satisfactory. Long-term stability is good if a reasonable warmup period is allowed; also, not too many jumps occur in its frequency. For O_3 we have used a Hewlett-Packard Model 200 oscillator, whose dial calibration is within ±2 percent of the true frequency. For precise work a better calibration would be desirable, but it was felt that in this case such a refinement was not worth while. One precaution worth mentioning is that the radio-frequency parts of the equipment must either be carefully shielded, or simply located so that it is not necessary for the operators to come near them during the performance of the experiment. The operators, unfortunately, introduce large variable capacitance.

In this experiment the student has a chance not only to see the utility of the general method but also to become acquainted with certain incidental experimental ideas of some importance. For instance, the device of taking a series of readings at various pressures, rather than one at each extreme, and plotting the results provides a way for the observer to keep an eye on the performance of the equipment, and allows him to correct for certain misbehavior. The student may also get an appreciation of the possibility of getting good results even if he has to take the readings, so to speak, "on the run," as he did in the slow leak method mentioned earlier. From a comparison of the results obtained by the two methods he can see the effect of the time element in certain types of measurement.

An experiment of this sort offers a number of lessons to the student. If he appreciates some of them he has benefitted from the experience.

Comments on the Optics of Birefringent Crystals

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(Received November 9, 1953)

Some of the optical properties of biaxial crystals are reviewed. Particular attention is given to the shapes of the wave surfaces and to the fact that true conical refraction cannot be observed.

COLLEGE texts on physical optics usually treat the subject of uniaxial crystals in some detail, but give inadequate attention to biaxial crystals. This is understandable since the biaxial case is considerably more complex, but unfortunate since so many minerals are biaxial. The purpose of this paper is to review some of the optical properties of biaxial crystals. The material presented here is not new, but the authors feel it has been neglected in many quarters. It is hoped that the paper will be of value to those who wish to pursue this subject somewhat beyond the usual college training and perhaps to those who care to review it.

Since the material is not new the emphasis will be placed upon the significance and interpretation of the established equations. Those who wish mathematical discussions are referred to earlier works.[1–3] The authors are not attempting to compete with these earlier writers but to present a brief resumé of their findings. It is assumed that the reader is familiar with the treatment of the subject as given in some textbook[4] for college juniors or seniors and that the usual notation can be used without detailed definition.

In order to establish the notation, as well as a train of thought, let us review briefly the fundamental ideas against the background of electromagnetic theory. The velocity of light in any transparent medium is determined by the dielectric constant of that medium. In birefringent crystals the dielectric constant is not a constant but is a function of the direction of the electric field. The dielectric constant is, therefore, a tensor relating the electric induction $\mathbf{D}$ to the field $\mathbf{E}$. These two vectors $\mathbf{D}$ and $\mathbf{E}$ are in general not parallel, but for a certain set of three mutually perpendicular directions $\mathbf{D}$ is parallel to $\mathbf{E}$. These three directions are called the *principal axes* of the crystal and are taken as the x, y, and z coordinate axes. The dielectric constants in these three directions are called the *principal dielectric constants* of the crystal and denoted by $K_1 = D_x/E_x$, $K_2 = D_y/E_y$ and $K_3 = D_z/E_z$. The three velocities v_1, v_2, and v_3, which are $c/\sqrt{K_1}$, $c/\sqrt{K_2}$, and $c/\sqrt{K_3}$, represent the velocity of light having its electric vector in the X, Y, and Z directions, respectively. It is convenient to so letter the principal axes that $v_1 < v_2 < v_3$ (i.e., $K_1 > K_2 > K_3$). The uniaxial crystal is a particularly simple case which results if v_2 is the same as either v_1 or v_3.

For any given direction of the normal to the wave front there are two values of the normal velocity v and these two velocities correspond to polarizations having mutually perpendicular $\mathbf{D}$ vectors. Likewise for any given direction of the ray velocity V and these will correspond to polarizations having mutually perpendicular $\mathbf{E}$ vectors. In any experimental setup, one may specify either the ray direction or the normal direction, but if one is fixed, the other is determined by the crystal. In general, the ray direction and the wave normal are not parallel, but there are exceptions when the electric vectors $\mathbf{D}$ and $\mathbf{E}$ are along the principal axes (i.e., parallel).

Problems in crystal optics are usually considered with reference to the *wave surface* which is also called the *ray surface*. The shape and properties of this surface are of interest. The surface is obtained by imagining an optical dis-

* Now at the Eastman Kodak Company, Rochester, New York.

[1] Max Born's *Optik* (Julius Springer, Berlin, 1933), Chap. V.

[2] W. Von Voigt, Physik Z. 6, 673 (1905); also Physik Z. 6, 818 (1905).

[3] *Handbuch der Physik*, Vol. XX (Julius Springer, Berlin, 1928).

[4] E.g., F. A. Jenkins and H. E. White, *Fundamentals of Optics* (McGraw-Hill Book Company, Inc., New York, 1950).
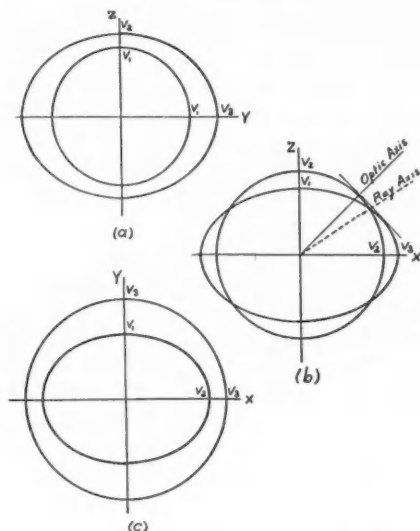
FIG. 1. The traces of the wave surface on the three coordinate planes.

turbance to spread out in all directions from a point O within the crystal. In each direction there will be two *ray* velocities. For each direction there are laid out two position vectors **r** starting at O and equal in length to the two ray velocities in that direction. The ends of the vectors then represent positions reached in unit time by the disturbance which started out from O. When all directions are considered, the end points of **r** form a surface. This is the ray surface, which is a double surface. It is represented by the following equation:

$$v_1^2(v_2^2-r^2)(v_3^2-r^2)x^2+v_2^2(v_3^2-r^2)(v_1^2-r^2)y^2 + v_3^2(v_1^2-r^2)(v_2^2-r^2)z^2=0. \quad (1)$$

In this form the equation appears to be of sixth degree but upon expansion may be divided by r^2 reducing it to fourth degree.

This surface is particularly useful because it enables one to determine graphically the direction of the wave normal associated with any given ray direction. A line is drawn from the origin O parallel to the given ray direction. This line pierces the wave surface in two points and the normals to the surface erected at these two points are parallel to the two wave normals which may be associated with the original ray direction.

The reverse process may be used to find the ray directions associated with a given wave normal.

Before considering the shape of this surface, the reader should remember that for birefringent substances the difference (v_3-v_1) rarely exceeds 10 percent of the average value of v_1, and v_3, and is usually only a few percent. The figures which are shown were drawn for the case $v_1=8$, $v_2=10$, $v_3=12$ and so represent an exaggeration of the actual effects. The trace of the ray or wave surface on each of the coordinate planes is a circle and an ellipse with the two "intersecting" only in the xz plane. Figure 1 shows these traces and indicates the graphical determination of the optic and ray axes which lie in the xz plane. At this point one must fit these three cross sections together to get a mental picture of the complete surface. The natural inclination is to use figures of revolution, but this is impossible. To aid in this difficulty some texts put the three traces together as in Fig. 2. Some more advanced texts provide a photograph of three-dimensional models of the wave surface as in Fig. 3. These models, made of plaster of Paris, are available and are of considerable value in visualizing the wave surface.

The models have one serious objection; they are always made with cuts which reveal only the planes of the principal axes. This clearly shows the similarity of the model and the cross sections of Fig. 1, but additional cuts would be helpful. For example, considerable point may be made of the fact that the outer and inner sheets of the wave surface meet at only four points. These four points lie in the xz plane and determine the
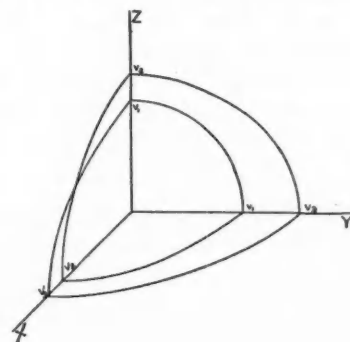
FIG. 2. One octant of the wave surface.
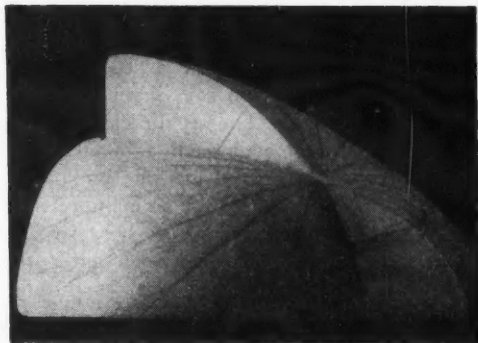
FIG. 3. A plaster model of the wave surface. [Reproduced from Georg Joos, *Theoretical Physics* (G. E. Stechert and Company, New York, 1934), by permission.]

two ray axes. It is not easy to imagine two sheets which can "intersect" so cleanly in Fig. 1(b) and yet have only point intersections, i.e., no curves of intersection. The two sheets do not intersect in the sense that two cylindrical surfaces often do, but like two wide angle cones placed one above the other and having their apexes in contact. It will help to visualize this situation to examine the trace of the wave surface on a plane parallel to the xz plane but slightly removed from it. Figure 4 is such a trace on the plane $y=1$. From this figure it is clear that the two sheets do not meet in this plane. The figure is almost a circle and an ellipse, but instead of crossing, the outer part of the circle joins the outer part of the ellipse; likewise, the inner parts join so that two separated sheets are formed. If we let z_o and z_i represent the z co-

ordinates of the outer and inner traces at that value of x for which the two traces have minimum separation in z, then

$$z_0 - z_i = \left(\frac{[5(v_2{}^2 - v_1{}^2)(v_3{}^2 - v_2{}^2)]^{\frac{1}{4}}}{v_3(z_0 + z_i)} \right) y. \quad (2)$$

For small values of y the sum, $z_o + z_i$, is approximately constant so that the separation of the two traces is proportional to y, the distance of the trace plane from the xz plane.

Since Eq. (1) contains only even powers of the coordinates it is apparent that the wave surface is symmetrical about the origin, the three coordinate axes, and the three coordinate planes. To make this point clear consider Fig. 5. This
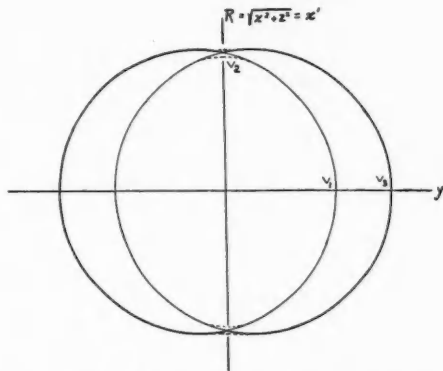
FIG. 5. The trace of the wave surface on the plane determined by the y axis and one of the ray axes.

is the trace of the wave surface on the plane determined by the y axis and one of the ray axes. The trace of this plane on Fig. 1(b) would coincide with the ray axis drawn there. At first glance this appears to be two circles neither of which is symmetrical about the origin or the line $y=0$. The curves are not circles since the diameter in the y direction is 20.0 and the diameter in the perpendicular direction 20.5. If one considers the inner sheet which consists of the shorter arcs of the two "circles," then this sheet is symmetrical as required. Likewise, the outer sheet, shown in heavy lines, is symmetrical. The trace shown here was obtained by rotating the x and z coordinate axes about the y axis by an angle $\tan^{-1}[v_1(v_3{}^2 - v_2{}^2)^{\frac{1}{2}}/v_3(v_2{}^2 - v_1{}^2)^{\frac{1}{2}}]$ and then plotting x' (the new x) against y for $z'=0$.

FIG. 4. The trace of the wave surface on a plane parallel to the xz plane, but slightly displaced from it.

FIG. 6. An experimental arrangement for demonstrating external double and conical refraction.

If the angle of rotation had been slightly different from the value specified, the inner and outer portions of this curve would not have touched but would have remained separate as suggested by the dotted lines. Under these conditions the symmetry is immediately recognized.

It has been shown by Voigt[2] that if a tangent plane is laid on the outer sheet of the wave surface in such a manner as to cover the dimple and be tangent at more than one place, then this plane will be tangent in a continuous curve which is a circle. It is remarkable that fo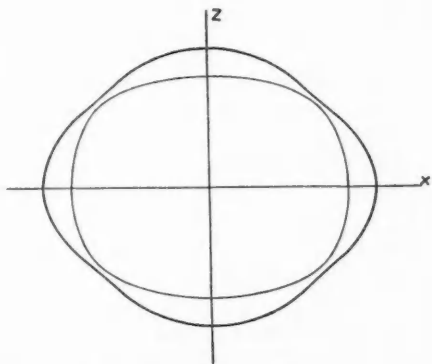r a surface apparently so complicated the tangent plane should touch in a continuous curve rather than in three or four isolated points. That this curve is a circle is even more remarkable. The rays from the origin through the circle of tangent points form a hollow cone and have their wave normals parallel to the optic axis. One of these rays is along the optic axis. The optic axis is the direction of a line from the origin perpendicular to the tangent plane. The cone is not a right circular cone since its circular base is perpendicular to one element of the cone instead of the axis. The diameter of the circle of tangency is

$$(1/v_2)[(v_2{}^2 - v_1{}^2)(v_3{}^2 - v_2{}^2)]^{\frac{1}{2}} \qquad (3)$$

and in the xz plane the apex angle of the cone is

$$\tan^{-1}(1/v_2{}^2)[(v_2{}^2 - v_1{}^2)(v_3{}^2 - v_2{}^2)]^{\frac{1}{2}}. \qquad (4)$$

We have been considering a cone of rays associated with the single normal along the optic axis. In like manner there is a cone of normals at the apex of the dimple. These normals are associated with a single ray direction along the ray axis. The ray axis is one element of this cone and the cone has an apex angle in the xz plane of

$$\tan^{-1}[\{[v_2{}^2 - v_1{}^2)(v_3{}^2 - v_2{}^2)\}^{\frac{1}{2}}/v_1 v_3].$$

This cone is not a right circular cone but has a circular section when cut by a plane perpendicu-lar to the cone element which is diametrically opposite to the ray axis.

As was mentioned earlier it is possible to fix experimentally either the ray direction or the wave normal direction. The ray direction may be fixed by an arrangement such as sketched in Fig. 6. The slab of crystal has one pinhole on each side of it. The upper pinhole is illuminated by convergent light or by diffuse light so that rays from a number of directions enter the crystal. The only light which emerges through the lower pinhole is that for which the rays are along the line $S_1 S_2$. For this ray direction there will usually be two directions of the wave normal determined by the normals to the wave surface where the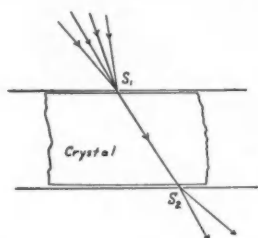 line $S_1 S_2$ pierces the inner and outer sheets. Since Snell's law of refraction applies to the normals and not to the rays, the light upon emerging at S_2 will form two separate rays in the air. This may be called *external double refraction*. For the special case in which $S_1 S_2$ is parallel to the ray axis there are, as we have seen, a cone of normals formed at the apex of the dimple and the light emerges from S_2 to form a hollow cone of light in the air. This is *external conical refraction*.[5]

The normal direction may be fixed by the arrangement shown in Fig. 7. Here the two pinholes S_1 and S_2 are on the same side of the crystal. The light in passing through them has its ray and wave normal directions fixed in the air. (In air the ray and wave normal are parallel.) The normals obey Snell's law upon entering the crystal and so are fixed in the crystal. The usual graphical construction based on Huygen's principle will determine the two refracted rays which
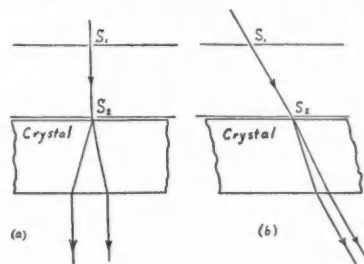

FIG. 7. An experimental arrangement for demonstrating internal double and conical refraction.

[5] For a laboratory experiment in conical refraction see J. R. Collins, Am. Phys. Teacher 7, 409 (1939).

diverge out from S_2 into the crystal. It is worth noticing that since the normals obey Snell's law they are confined to the plane of incidence, whereas the rays are not so confined and usually depart from the plane of incidence. (For the general case the two normals are not exactly parallel since the two normal velocities in the crystal are not exactly equal. If it is desired to have the normals exactly parallel the light must fall normally on the surface of the crystal *or* be refracted along the optic axis.) If the crystal is in the form of a parallel plate the two rays will continue to separate until they reach the second face, at which place they will be refracted to follow their original direction. This may be called *internal double refraction*. For the special case in which the wave normal lies along the optic axis there will be a hollow cone of rays in the crystal and a cylinder of rays emerging from the lower face of the crystal. This is *internal conical refraction.*

This very brief review of conical refraction would be rather pointless except to set the stage for a question which the more discerning reader will wish to ask. If each of the two cases of conical refraction takes place only when the ray (or wave normal) is in a mathematically unique direction how then can any finite energy be associated with conical refraction. Conical refraction is an experimental reality, yet the theory predicts that any deviation (no matter how small) from the specified direction will produce double refraction not conical refraction. This difficulty was resolved by Voigt.[2] It will be discussed here for the case of internal conical refraction. The external case is slightly more involved but follows along the same general lines.

The two pinholes in Fig. 7 are of finite size and so limit the light to a narrow cone of wave normals but not to a single normal as discussed in the theory. It is proper, therefore, to ask what happens to the refracted light for which the normals deviate only slightly from the optic axis. For the internal case this involves the shape of the wave surface in the immediate vicinity of the circle of tangency which was described previously. The plane is tangent to the surface along this circle but if one departs from the circle the plane and the surface separate. It is very much like a toroid with a plane surface lying on it.

(Obviously this analogy is to be used only for points near the circle of contact, but there the toroid is a good approximation.) If the wave normal departs slightly from the optic axis there will be only two points on the toroid for which the surface is perpendicular to the specified normal; these will be very close to the circle. One will be just inside the circle and the other diametrically opposite but just outside the circle. For example, consult Fig. 8 which is an exaggerated drawing of the surface near the circle of contact. The direction of the optic axis is *oa*. The normals to the surface will be parallel to *oa* if constructed at any point along the circle of contact. Four such normals are shown at A, A', A'', and A'''. For directions such as *ob*, which
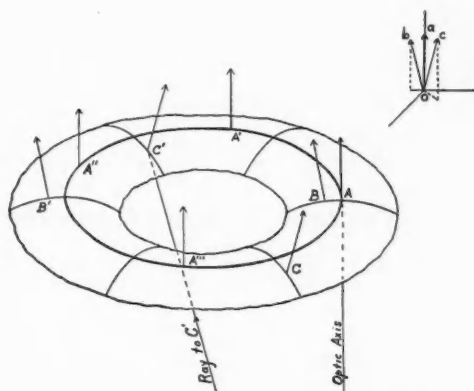
FIG. 8. An exaggerated drawing of that portion of the wave surface which surrounds a dimple.

are not quite parallel to the optic axis, there will be only two places on the surface which are perpendicular to *ob*. These are at B and B'. Likewise C and C' are the only points for which the surface is perpendicular to *oc*. The two corresponding rays are from the origin through points C and C'.

From this it may be concluded that the infinitesimal amount of energy having its wave normal parallel to the optic axis is distributed among the infinite number of points along the circle $AA'A''A'''$ and the resulting intensity at the circle is zero. On the other hand, the infinitesimal energy corresponding to the wave normal *oc* emerges as the two rays through the
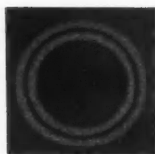
FIG. 9. Conical refraction, showing the dark ring. (Reproduced from Max Born, *Optik*, copyright 1933 by Julius Springer, Berlin, Germany. Reproduced by permission of the Attorney General of the United States.)

two *points* C and C' and the intensity is finite as in every other case of double refraction. The narrow cone of normals surrounding the optic axis on all sides thus produces a composite of points such as C, C', B, and B'. This composite of points produces two bright rings one within the circle of contact and the other outside, but the circle of contact is itself *dark*. Thus it has been shown that the experiments in conical refraction produce an unusual case of double refraction; but conical refraction, as usually explained, cannot be observed. The dark ring was first reported by Poggendorff[6] in 1839 and Haidinger[7] in 1853. However, it was not until 1905 that any explanation was published.

The intensity of the two concentric rings rises from zero as a linear function of the distance from the circle of true conical refraction. This is readily seen since the distance of points such as C and C' from the circle is proportional to the angle θ between the wave normal and the optic axis. Also the energy flow included between θ and $\theta+d\theta$ is proportional to θ. Since the rings are thin compared to their radii it follows that the intensity is proportional to the distance from the circle of true conical refraction. This linear increase ends abruptly and the intensity falls to zero when θ becomes large enough to have included all of the normals in the narrow cone. This maximum value of θ is determined by the size and separation of the pinholes S_1 and S_2. The actual appearance of this ring is shown in Fig. 9.

[6] J .C. Poggendorff, Pogg. Ann. **48**, 461 (1839).

[7] W. Haidinger, Pogg. Ann. **86**, 486 (1853).

The Abbe Theory of Microscopic Vision and the Gibbs Phenomenon

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(Received December 16, 1952)

An elementary discussion of the Abbe theory of microscopic vision is given, suitable for teaching purposes. The effect of the phases in the back focal plane of the objective is briefly outlined. The consequence of limiting the aperture of the system in this plane is shown to be an image marred by diffraction.

The relationship to Fourier's theorem having been indicated, the Gibbs phenomenon is next described. A standard formula for the finite sum of a Fourier series is taken and adapted to the case of imaging a suitable grating with a microscope objective limited by a slit aperture. If the grating slits are very narrow compared with the opaque region separating them, the image of each is shown to be marred by a set of fringes, due to the Gibbs phenomenon, exactly equivalent to the diffraction fringes from the single slit in the microscope objective.

THE Abbe theory of microscopic vision is of historical interest to all teachers of optics. For, though we seldom nowadays use a microscope under precisely the conditions it envisages, it has thrown much light on Fraunhofer diffraction and image formation and leads indirectly to such developments as Fourier synthesis in x-ray crystallography, and by analogy, to certain problems in transmission lines.

It is as well to begin with a brief resumé of the Abbe theory, and the conditions under which it applies. We assume, first, that the object is transilluminated from a point source or fine aperture at the focal point of a condenser, thus giving a parallel beam of coherent illumination (Fig. 1).[1] The object alters the condition of the

[1] For a full discussion of coherence conditions, see H. H. Hopkins, Proc. Roy. Soc. (London) **A208**, 263 (1951). The Hopkins criterion is probably rather strict for practical use, as it reduces the effective beam intensity considerably. Gabor, Proc. Phys. Soc. (London) **B64**, 449 (1951) uses the Airy disk as the criterion, which is 3.83 times as large in diameter as Hopkins' critical aperture. A rough working compromise might be 3 times the Hopkins limit.

beam. In particular, if it is a repeated structure or grating, it gives rise to a series of lateral beams according to the grating equation. The whole assemblage (or as much of it as possible) is collected by the objective, and the lateral beams are focused into a series of lateral spectra in the back focal plane of the objective. In the general case, where the object is not a repeated structure, the pattern formed in the back focal plane of the objective is the Fourier transform of the object, up to a certain order, with certain phase terms added.

Next, we may regard the system in the back focal plane of the objective as a set of coherent sources, with some known phase relationship, capable of giving rise to a complicated Young interference pattern throughout the rest of space. At first sight, *any* plane in space beyond the back focal plane seems equally likely to represent the object, yet it can be shown that only one such plane gives a true representation, owing to the phase relationships between the spectra.

Strictly speaking, a particular isolated wavelength should be used for this work, and this is easily obtained in experimental setups. But this limitation has been considered carefully by Conrady[2] who has shown qualitatively that one of the distinguishing features of the image plane is the absence of any variation *in this plane*, if the wavelength is varied. He has further shown that the pattern in the image plane is insensitive to lateral movement of the source in the focal plane of the condenser.[3] Thus Conrady has shown that, in practice, the use of white light and extended source contributes greatly to the recognition of the plane of correct focus. Conversely, Porter[4] has shown that if the wavelength is held constant and if the source is very small and the object is a strict grating, the "image plane" is no longer distinguishable within an extended three-dimensional region. In fact, the pattern in any one plane is as good a reproduction of the object as the pattern in any other plane. The extensions of Conrady are of great practical importance, as they enable the Abbe theory to be used qualita-
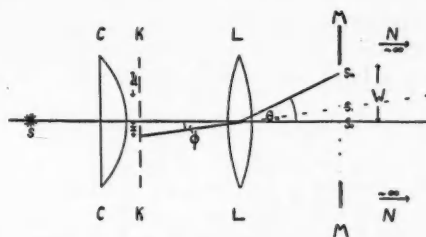
FIG. 1. Microscopical arrangement envisaged in the Abbe theory. S a small or point source in the front focal plane of the condenser CC, giving a parallel coherent incident beam of light, falling on the object KK, which is here a grating situated at or just outside the front focal plane of the objective lens LL. The grating gives rise to a set of spectra $S_0, S_1, \cdots S_n$ in the back focal plane MM of the lens LL. It will be noted that S_0 is an image-point conjugate with S in the combination of lenses CC and LL: the other spectra may be regarded as diffraction images of S, and if S is small and finite, they will all have the shape of S. The system of spectra in the plane MM may now be regarded as a coherent system of sources, any two of which may be paired to give an elementary system of Young interference fringes. The combination of these Young interference systems in the image plane NN (off the diagram to the right and here nearly at infinity) give rise to a representation of the object by a form of Fourier synthesis. W is the half-width of a slit in the plane MM; θ_n the angular coordinate of the nth spectrum; d the grating spacing, and x, φ are linear and angular coordinates of a point in the image, referred back for convenience to the geometrically conjugate point in the object plane KK.

tatively even on systems of wide condenser aperture, though they do complicate quantitative calculations of the errors involved in such systems.

So far, there has been no discussion of possible errors in image formation. So long as the number of spectra in the back focal plane is sufficiently large, or the Fourier transform is virtually complete, the errors in image formation are negligible. But as Abbe pointed out, errors are bound to arise if the number of spectra transmitted is small. Thus unless the zero-order spot and at least one first-order spectrum is transmitted, a grating cannot be resolved at all, and even if this condition is fulfilled the image will be a sinusoidal variation of light, whatever the profile or black/white ratio of the grating: the only thing which may vary in the image is the depth of indentation of the background by the sinusoidal variation. The image is thus very imperfect.

It will be noted that the zero-order spot is not necessarily central (corresponding to an illuminating point off the axis of the condenser), and thus the objective may accept the zero-order and one lateral first-order spectrum, with-

[2] Conrady, J. Roy. Microscop. Soc. 626–30 (1904).
[3] The Hilger Nonrecording Microphotometer uses this property as a test of the accuracy of focus. The source is effectively moved, and if the image does not move with it, the image is correctly focused.
[4] Porter, Phys. Rev. 24, 303 (1907).

out the corresponding first-order spectrum on the other side. This is, indeed, the main advantage of using a large illuminated aperture in the condenser.

Abbe, however, went further. He not only formulated the minimum condition for resolution given above, but he saw that the image arises, *inter alia*, from the spectra passed through the back focal plane. This theory may be formulated in its most general form: The image formed in a microscope corresponds exactly to that hypothetical object which gives rise to just the spectra actually passed by the objective and no other spectra.[5] This formulation is, admittedly, not entirely unambiguous, since in certain cases two objects can give rise to sets of spectra visually indistinguishable; in fact, they differ in phase and not in intensity. The possibilities of error have been considered by a number of crystallographers;[6] the general result is that the more complex the object, the less the likelihood of ambiguity.

The question of the phases of the various spectra in the back focal plane of an objective viewing a repeated structure has aroused considerable interest and discussion, and given rise to various ideas, not always strictly accurate. It is therefore of some value to run briefly through the salient features.

The fact is, that the pattern in the back focal plane of the objective is only *strictly* a Fourier transform of the object, i.e., correct in both amplitude and phase, when the object is in the front focal plane of the objective—a fact not normally stated.[7] If, now, the object be displaced laterally in the front focal plane, the effect on the spectra of this lateral displacement is to introduce a linear variation of phase with order in the back focal plane. This linear variation of phase is equivalent to a linearly varying retardation on one side, corresponding to a thin prism. The center of the complicated Young interference pattern is thus laterally displaced.[8]

On the other hand, if the object is simply displaced along the axis, the result on the phases in the back focal plane is to impose a circularly symmetrical variation of phase on the previous system. The phase shift introduced at any point in the back focal plane is proportional to the square of the distance of this point from the axis. This corresponds to a thin lens. The effect on the complex Young interference pattern is to shift the plane at which it reconstructs the object correctly, i.e., to "focus" it.[8]

The next stage in the development of the subject came with the explicit realization that Fourier's theorem had an application to optics. Rayleigh[9] in an interesting paper proved that the total light in all possible spectra from a diffraction grating adds up to the transmitted fraction of the incident light. To do so, he employed a Fourier series and by implication identified the amplitudes in the various orders of diffraction with the coefficients of the series. This was in 1874. By 1896, the use of Fourier series had become so all-pervading in Rayleigh's works[10] that it is clear that he knew of their importance in diffraction.

In 1904, Everett[11] explicitly linked the formation of the image of a grating with a Fourier series. In 1906 Porter,[12] in an important paper, examined in detail the way in which the image will change with the number of spectra passed by the objective. The problem is treated by taking the Fourier expansion of the grating profile and examining the result of using 1, 2, 4, 5, 7, 8, 10, 11 terms in the synthesis, corresponding to this number of spectra each side of the central zero-order spot in the back focal plane of the examining objective. The curves he gives in his Fig. 6 are at once reminiscent of diffraction effects, and agree with his practical observations. He shows, further, than an increase of aperture *may* result in a more confusing image than that with one spectrum each side only. By now, diffraction and incomplete Fourier series are closely connected.

[5] See, e.g., R. W. Wood, *Physical Optics* (Macmillan Company, New York, 1905), 2nd edition, p. 181.
[6] E.g., A. L. Patterson, Phys. Rev. **65**, 195 (1944).
[7] But see an important recent paper by J. Elmer Rhodes, Am. J. Phys. **21**, 337 (1953), where the fact is explicitly stated and developed.
[8] A very full discussion of these two points is given in H. H. Hopkins *Wave Theory of Aberrations* (Clarendon Press, Oxford, 1950), pp. 12–19.

[9] Lord Rayleigh, Phil. Mag. **47**, 81–93, 193–205 (1874); *Collected Works* (Cambridge University Press, Cambridge) Vol. I, p. 199.
[10] Rayleigh, Phil. Mag. **42**, 167 (1896); *Collected Works* (Cambridge University Press, Cambridge) Vol. IV, p. 250.
[11] J. D. Everett, J. Roy. Microscop. Soc. 385 (1904). This paper contains an obvious misprint.
[12] Porter, Phil. Mag. Ser. 6, **11**, 154 (1906).

It is not clear whether Abbe had an explicit knowledge of this connection before his death in 1905, though his theory carries it implicitly. In 1910, Lummer and Reiche[13] published his lecture notes, which book contains a very full and explicit discussion, together with acknowledgments of the above two papers.

Porter's work is of historical interest because it led the Braggs[14] to use Fourier synthesis in x-ray crystallography.

The general idea of analyzing the process of image formation into two stages has been further developed in recent times. In one group of methods, analysis is in effect into two stages of Fraunhofer diffraction, involving a double Fourier transform.[15-19] These methods have been mainly used by crystallographers, in an attempt to obtain crystal structures by a combination of x-ray and optical methods. They are of particular interest in that the wavelength λ is changed in the middle of the process with a corresponding considerable magnification.

In another and even more general idea the process can be analyzed into two suitably related *Fresnel* diffraction processes, again with a change of wavelength. This is the basic idea behind Gabor's diffraction microscope, which has also been considerably developed of late.[20-24]

We also have, following Porter's work, a number of workers who use the double Fourier transform idea, with truncated transforms, to give analytical or numerical information about diffraction effects with finite or limited objectives.

Among others we may cite Duffieux,[25] Hopkins,[26] and others.[27]

Out on the other wing, the mathematicians were examining the Fourier series *per se*. Thus in 1898, Gibbs published two short letters in *Nature* which drew attention to a peculiarity of the series.[28] If a wave be analyzed which has a vertically rising edge (e.g., a square-topped or saw-toothed wave), and then a synthesis be made, it is found that the vertical edge is replaced by a very rapidly rising front, which "overshoots" the mark: for a saw-toothed wave by 0.17898 of the correct amplitude. This holds even if the number of terms is indefinitely increased, provided that the curve we are studying is regarded as the limiting curve of a family of curves of increasing perfection. But if we go to the limit of the *series*, at each point, point by point, we get a table of values corresponding to the original. Thus the result obtained varies according to the way we proceed to the limit. This "overshooting" is called the "Gibbs phenomenon" (Fig. 2).

In course of time, the usage of this term has
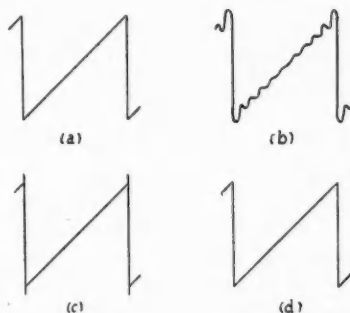
FIG. 2. The Gibbs phenomenon. (a) Original saw-toothed wave; (b) Incomplete synthesis, with 10 terms; (c) Result of taking a series of continuous curves of incomplete syntheses and noting how their shape changes as the number of terms used increases indefinitely; (d) Result of calculating, point by point, the local value of $f(x)$, using the infinite Fourier series, and then plotting the results. Note that there are *two* ways of proceeding to the limit: one can draw a continuous curve and allow it to proceed to the limit, giving 2(c). Or one can proceed to the limit of the series *at each point*, and *then* draw a curve 2(d). The two results are *not* equivalent.

[13] Lummer and Reiche, *Bildentstehung im Mikroskop* (Leipzig, 1910).

[14] W. H. Bragg, Phil. Trans. Roy. Soc. **A215**, 253 (1915).

[15] H. Boersch, Z. techn. Phys. **19**, 337 (1938).

[16] W. L. Bragg, Nature **143**, 678 (1939); **149**, 470 (1942).

[17] M. J. Buerger, Proc. Natl. Acad. Sci. **25**, 383 (1939); **27**, 117 (1941); **36**, 330 (1950); J. Appl. Phys. **21**, 990 (1950).

[18] H. Lipson *et al.*, Acta Cryst. **4**, 261, 458 (1951); **5**, 145, 362 (1952).

[19] R. W. James, *The Crystalline State* (G. Bell and Sons, London, 1948), Vol. II, pp. 385–403.

[20] Gabor, Nature, **161**, 777 (1948); Proc. Roy. Soc. (London) **A197**, 454 (1949); Proc. Phys. Soc. (London) **B64**, 449 (1951).

[21] M. E. Haine and T. Mulvay, J. Opt. Soc. Am. **42**, 763 (1952).

[22] M. E. Haine and J. Dyson, Nature **166**, 315 (1950).

[23] W. L. Bragg, Nature **166**, 399 (1950); **167**, 190 (1951).

[24] G. L. Rogers, Proc. Roy. Soc. (Edinburgh) **63**, 193, 313 (1952).

[25] P. M. Duffieux, Ann. phys. **19**, 363 (1944); Compt. rend. **220**, 846 (1945); **222**, 1482 (1946).

[26] H. H. Hopkins, Sci. J. Roy. Coll. Sci. (London) **20**, 1 (1951).

[27] B. A. N. Caspersz, quoted by Hopkins in reference 26 above.

[28] J. W. Gibbs, Nature **59**, 200, 606 (1898). *Collected Works* II, p. 258.

widened, and it is now applied to any difference between the actual Fourier sum and the original, particularly those due to the use of a finite series of terms. This particular subject has been exhaustively studied by many workers.

To illustrate the connection, we shall take a one-dimensional case, and consider a grating consisting of comparatively narrow transmitting lines separated by comparatively wide dark or opaque spaces. It is our ultimate intention to allow the dark spaces to get indefinitely large, so that we are in fact imaging an indefinitely fine slit by an objective which we shall stop down laterally. In this way, we shall get the classical result for the Fraunhofer pattern due to a slit stop, on a point source or a line source parallel to the slit.

If $f(x)$ be a function depending on a linear variable, x, which gives the amplitude transmission coefficient in the line $x=x$ in the x, y plane (object plane), we can find the result of taking a Fourier sum to n terms only, $S_n(x)$, without explicitly evaluating the $2n+1$ Fourier coefficients, by the use of the formula of Jeffreys and Jeffreys:[29]

$$S_n(x) = \frac{1}{2\pi} \int_0^{2\pi} f(t) \frac{\sin(n+\frac{1}{2})(t-x)}{\sin\frac{1}{2}(t-x)} dt, \quad (1)$$

a recognized expression for the Gibbs phenomenon.

Turning now to Fig. 1, we see that we are using a black and white grating so that $f(x)=0$ in the black regions and $=1$ in the white regions. We take our origin in the middle of a white region, and assume that the white region has a width of $2a$ "units," where the unit of length is here specially chosen so that the repeat distance, d, of the grating is numerically equal to 2π units. We then have a formal statement of $f(x)$ in the following terms:

$$f(x)=1 \text{ if } -a<x<a; \ 2\pi-a<x<2\pi+a;$$
$$2n\pi-a<x<2n\pi+a$$
$$f(x)=0 \text{ if } a<x<2\pi-a; \ 2(n-1)\pi \quad (2)$$
$$+a<x<2n\pi-a.$$

It is convenient to leave $f(x)$ undefined *at the*

[29] Jeffreys and Jeffreys, *Mathematical Physics* (Cambridge University Press, Cambridge, 1946), pp. 403, 416, Sec. 14.04, Eq. (3) and Sec. 14.07.

points $\pm a$ etc., as it can never be exactly represented here by the Fourier series in any case.

It will further be noticed from Fig. 1 that we have placed the grating object close to the front focal plane of the objective, as is usual in microscopical observation. The focal length of the objective is F. For simplicity, we shall take the stop to be in the back focal plane of the objective, when it will either pass or not pass a given order of spectrum. If it is placed elsewhere, e.g., in the objective itself, it will simply result in a shaded cutoff of the higher-order spectra, with additional complication to the analysis. The stop we take to be slit shaped, parallel to the grating slits, and of half-width W. We shall arrange W to make the edge of the slit lie half-way between the nth and the $n+1$th spectra, thus clearly passing the nth and equally clearly cutting off the $n+1$th.

If now θ_n is the *angle* at which the nth spectrum is formed and d is large compared with the wavelength λ, we find that θ_n is small and approximates to $\sin\theta_n$. Thus we may approximate to the grating formula

$$n\lambda = d\theta_n. \quad (3)$$

But the *linear* distance from the axis in the back focal plane is clearly $F\theta_n$ or $nF\lambda/d$.

Since W corresponds to the "$n+\frac{1}{2}$"th spectrum, we have:

$$W = (n+\frac{1}{2})F\lambda/d = (n+\frac{1}{2})F\lambda/2\pi, \quad (4)$$

when λ is measured in the same units as a and d (when $d=2\pi$, as substituted) and W and F are both measured in a common set of units, not necessarily that for a, d, and λ.

Further, to eliminate F, we may measure x in angular coordinates so that $x=F\varphi$. This then gives

$$W = (n+\frac{1}{2})x\lambda/2\pi\varphi, \quad (5)$$

where W and F must now be related to the common unit for λ, a, and d.

Returning now to our finite sum if we substitute the value of $f(x)$ defined by Eq. (2) in our formula (1), we get

$$S_n(x) = \frac{1}{2\pi} \int_{-a}^{+a} \frac{\sin(n+\frac{1}{2})(t-x)}{\sin\frac{1}{2}(t-x)} dt, \quad (6)$$

where the limits have been altered to restrict the

integration to the interval in which $f(x)$ is not zero.

In the analysis to follow, we are first going to limit the value of a. This corresponds to a grating which is largely opaque, having only very narrow transparent slits in it. In practice, we might make them a few microns wide. By making d much larger, we can keep it large compared with the wavelength λ as required by our approximate Eq. (3). We shall then consider what happens if we allow d to increase while keeping a fixed, thereby still further reducing the fraction of light passed by the grating and also reducing the fraction of a period for which $f(x) = 1$. As d increases, we increase the number of orders n passed by the slit W, but we find that we do not thereby transmit more information about the slit a. For as the ratio a/d becomes smaller, the higher order terms in the Fourier series become more and more important, and many of these are still excluded by the slit W.

In order to simplify the integral [Eq. (6)], we next assume that a is very very small, so that we may replace the integral by $2a \times$ (height of the ordinate at $t = 0$). This gives:

$$S_n(x) \simeq \frac{a}{\pi} \cdot \frac{\sin(n+\frac{1}{2})x}{\sin\frac{1}{2}x}. \tag{7}$$

Our next step is to take x so small that $\sin x \simeq x$;

but *not* necessarily so small that $\sin(n+\frac{1}{2})x \simeq (n+\frac{1}{2})x$. We then get

$$S_n(x) \simeq \frac{a}{\pi} \frac{\sin(n+\frac{1}{2})x}{\frac{1}{2}x} = \frac{2a(n+\frac{1}{2})}{\pi} \frac{\sin(n+\frac{1}{2})x}{(n+\frac{1}{2})x}$$
$$= \frac{2a(n+\frac{1}{2})}{\pi} \frac{\sin u}{u}, \tag{8}$$

where $u = (n+\frac{1}{2})x.$ (9)

Now we have already seen that

$$W = (n+\tfrac{1}{2})x\lambda/2\pi\varphi, \tag{5}$$

and therefore,

$$u = (n+\tfrac{1}{2})x = 2\pi W\varphi/\lambda, \tag{10}$$

and we get

$$S_n(x) = \frac{2a(n+\frac{1}{2})}{\pi} \frac{\sin(2\pi W\varphi/\lambda)}{(2\pi W\varphi/\lambda)}, \tag{11}$$

a well-known formula for the diffraction pattern due to a slit.[30]

It is, of course, to be noted that it is the slit of half-width W which is doing the diffraction (by limiting the spectra), and the grating opening a is analogous to a fine object slit whose image is spoiled by diffraction. φ is an angular parameter in object *or* image space, and thus we may write the parameter u as

$$u = \frac{\pi \times \text{width of slit} \times \text{angular coord. of point of observation}}{\text{wavelength}} \tag{12}$$

a more familiar expression of the formula.

We see further from the factor $2a(n+\frac{1}{2})/\pi$ in front of the $\sin u/u$ term that the maximum amplitude depends on (i) the width of the initial (object) slit, and (ii) the width of the stop slit (which governs the order n).

It may be objected that we have a whole series of object slits in the grating and that these will interact. It will be observed, however, that Eq. (11) is independent of d (the absolute value of a being assumed constant), and hence we can let d become very large. Since x is in any case re-

stricted by the $\sin x \simeq x$ equivalence to a very small region compared with d, it is seen that the formula is valid only in the neighborhood of the geometrical image, and there gives adequate results provided d is very large.[31]

[30] See e.g., Airy, *Undulatory Theory of Optics* (Macmillan and Company, Ltd., London, 1877), p. 72. Preston, *Theory of Light* (Macmillan and Company, Ltd., London, 1901), third edition, pp. 259–60; Schuster and Nicholson, (Edward Arnold and Company, London, 1924), third edition, p. 103.

[31] The procedure of taking the limit $d \to \infty$ is difficult if not impossible to justify with strict rigor, especially as the coherence conditions become progressively more stringent.

Reproductions of Prints, Drawings, and Paintings of Interest in the History of Physics

62. The Invention of the Siphon

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(Received January 29, 1954)

Egyptian pictorial records dating from before 1100 B.C. exist which show siphons and drinking-tubes in use. Two of these are here reproduced, together with the explanation of the operation of siphons given by Heron of Alexandria in the first century A.D.

SIPHONS are clearly among the oldest of pneumatic devices. It is not known, however, when or where they were invented, or by whom. J. G. WILKINSON in his *Manners and Customs of the Ancient Egyptians* (London, 1837)[1] gives the drawing here reproduced as Fig. 1 and writes as follows:

"Siphons are shown to have been invented in Egypt, at least, as early as the reign of Amunoph II, 1450 years before our era; and they again occur in the paintings of the third Rameses. In a tomb at Thebes bearing the name of Amunoph, their use is unequivocally pointed out, by one man pouring a liquid into some vases, and the other drawing it off, by applying the siphon to his mouth, and thence to a large vase; and it is not improbable that they owed their invention to the necessity of allowing the Nile water to deposit its thick sediment in vases, which could not be moved without again rendering it turbid, whether by inclining the vessel, or dipping a cup into it with the hand." Thomas Ewbank in his *A Descriptive and Historical Account of Hydraulic and Other Machines for Raising Water*[2] reproduces Wilkinson's drawing and says: "The researches of Rosellini and Wilkinson . . . have brought to light irrestistible evidence that siphons were used in Egypt at least as early as 1450 years before Christ. In a tomb at Thebes, which bears the name of Amunoph II, who reigned at the period just named, they are delineated in a manner too distinct to admit of any doubts." Wilkinson's drawing and statement are also reproduced, usually without indication of provenience, in most of the standard histories of science or technology that deal with this subject[3] and his conclusions seem to have received universal acceptance.

Having learned by experience, however, that before accepting historical statements of this kind one must go back to the original sources, I wrote to the Oriental Institute of the University of Chicago to inquire whether the scene copied by Wilkinson is still in existence and whether modern researches would lend support to Wilkinson's conclusions. Dr. George R. Hughes very kindly replied that there are five private

FIG. 1. Egyptians siphoning off water or wine. (From *Manners and Customs of the Ancient Egyptians*, by J. G. Wilkinson (London, 1837).

[1] J. G. Wilkinson, *Manners and Customs of the Ancient Egyptians* (London, 1837), Vol. III, pp. 340–341.

[2] Thomas Ewbank, *A Descriptive and Historical Account of Hydraulic and Other Machines for Raising Water* (Appleton-Century-Crofts, Inc., New York, 1842), p. 516.

[3] E.g. E. Gerland and F. Traumüller, *Geschichte der Physikalischen Experimentierkunst* (Leipzig, 1899); A. de Rochas, *Les Origines de la science et ses premières applications* (Paris, 1883); Adolf Erman, *Aegypten und Aegyptischen Leben im Altertum* (Tübingen, 1885); M. N. Baker, *The Quest for Pure Water* (The American Waterworks Association, New York, 1949).

tombs at Thebes known to bear the name of an Amenophis (or Amunoph), but that the siphoning scene does not appear in any of them at present in so far as he could ascertain. Moreover, none of the tombs happens to date to the reign of Amenophis II. He thought, however, that Wilkinson did see the scene he described, but that it might have disappeared or been destroyed. Also the tomb might have been covered up and its whereabouts forgotten since Wilkinson's time, or he may have made a wrong attribution of it, a mistake he occasionally made. Dr. Hughes therefore wrote to Miss Rosalind Moss of Oxford, author of the *Topographical Biography of Ancient Egyptian Hieroglyphic Texts*, to see if she knew where the scene is to be found. Miss Moss replied that the scene was copied by Wilkinson from the tomb of a priest named Kynebu at Thebes (No. 113) who decorated his tomb in the reign of Ramses VIII (ca 1142–1138 B.C.). The reliefs of the tomb are now almost entirely destroyed and the scene is gone, but Miss Moss was able to locate it by use of the unpublished manuscripts of early copyists in the Griffith Institute, Asmolean Museum. It therefore seems safe to say that siphons were indeed *in use* in Egypt by 1100 B.C., but there is apparently no evidence that they were *invented* there.

A sounder analysis of the problem than that of Wilkinson is given by Albert Neuburger in *The Technical Arts and Sciences of the Ancients*,[4] who writes as follows: "A number of ancient representations inform us that the Egyptians not only transferred liquid from one vessel to another by means of siphons but also used them in drinking. The forerunner of the siphon may have been a pipe used for sucking as depicted in Fig. 2: such a pipe cannot be regarded as a true siphon although it made use of the atmospheric pressure on liquids to raise them out of a vessel to the level of the mouth. If, while sucking or drinking, the longer limb of a bent tube were allowed to fall quickly enough and if by accident the lower surface of the liquid in this limb came to lie below the free surface of the liquid in the vessel, siphoning would occur

FIG. 2. A seated Syrian sucking up liquid out of a jar by means of a bent tube. (From "Grabstein eines syrischen Söldners aus Tell Amarna," by W. Spiegelberg and A. Erman in Z. Aegypt. Sprache Altertumskunde, Vol. **XXXVI**, 126–129 (1898), Tafel XVII.)

automatically, and the liquid would flow out in spite of its having to flow upwards in the shorter limb. Perhaps Fig. 2 actually depicts how a siphon is started by sucking. The three points which favor this view are the length of the one limb (which is apparently being supported to prevent the snapping of the bent tube), the fact that there is a bend, and that the youth is holding in his left hand another vessel which is probably to be filled and offered to the woman waiting on the right."

Figure 2 is from a photograph of a painted tombstone in the form of a door in the Egyptian Section of the Berlin Museum (No. 14122). The original tombstone came from Tell el-Amarna and dates from about 1380–1360 B.C. Although the painting is in good Egyptian style, the soldier depicted is a Syrian (probably a mercenary) and F. L. Griffith[5] makes use of the scene to identify certain metal strainers and metal tubular angles (for joining the vertical and horizontal sections of such drinking tubes)

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[4] Albert Neuburger, *The Technical Arts and Sciences of the Ancients*, translation by Henry L. Bose (Methuen and Company, Ltd., London, 1930).

[5] F. L. Griffith, J. Egypt. Archaeol. **XII**, 22–23 (1926).

which were found at Ámarna and Tell el-Yahu-
diyeh in Egypt where Dr. Hughes informs me
foreign influence and eclectic tendencies might
be expected. Moreover, drinking tubes are
depicted on cylinder seals found in excavations
in Syria and Iraq[6] and there is a one-piece metal
drinking tube in the Oriental Institute at
Chicago which was found at Tell Asmar in
Iraq and which probably dates from about
2700 B.C.[7] All this would seem to suggest that
the drinking tube and perhaps the siphon were
invented in Mesopotamia much earlier than 1450
B.C. and that they were introduced into Egypt
during the second millennium B.C.[8]

Siphons of several different types and many
interesting and amusing applications of siphons
are described in the *Pneumatics* of Heron of

FIG. 3. The bent siphon. (From *The Pneumatics
of Hero of Alexandria.*)

[6] See H. Frankfort, *Cylinder Seals* (London, 1939),
pp. 77–78.
[7] H. Frankfort, *Iraq Excavations of the Oriental Institute,*
1932/33 (Oriental Institute Publication No. 17, Chicago,
1934), Fig. 35.
[8] See M. E. L. Mallowan in *Illustrated London News,*
March 27, 1937, p. 518.

Alexandria, written probably during the first
century A.D.[9] Since this work provides ones of
the earliest attempts to explain the operation
of a siphon, the description of the bent siphon
given in it will be quoted in full.[10]

"Let *ABC* [Fig. 3], be a bent siphon, or tube,
of which the leg *AB* is plunged into a vessel *DE*
containing water. If the surface of the water
is in *FG*, the leg of the siphon *AB*, will be filled
with water as high as the surface, that is, up to *H*,
the portion *HBC* remaining full of air. If, then,
we draw off the air by suction through the
aperture *C*, the liquid also will follow from the
impossibility, explained above, of a continuous
vacuum. And, if the aperture *C* be level with the
surface of the water, the siphon, though full,
will not discharge the water, but will remain
full: so that, although it is contrary to nature
for water to rise, it has risen so as to fill the tube
ABC; and the water will remain in equilibrium,
like the beam of a balance, the portion *HB* being
raised on high and the portion *BC* suspended.
But if the outer mouth of the siphon be lower
than the surface *FG*, as at *K*, the water flows out;
for the liquid in *KB*, being heavier, overpowers
and draws toward it the liquid in *BH*. The dis-
charge, however, continues only until the surface
of the water is on a level with the mouth *K*,
when, for the same reason as before, the efflux
ceases. But if the outer mouth of the tube be
lower than *K*, as at *L*, the discharge continues
until the surface of the water reaches the mouth
A. If then we wish all the water in the vessel to
be drawn out, we must depress the siphon so far
that the mouth of *A* may reach the bottom of the
vessel, leaving only a passage for the water.

"Now some writers have given the above ex-
planation of the action of the siphon, saying that
the longer leg, holding more, attracts the shorter.
But that such an explanation is incorrect, and
that he who believes so would be greatly mis-
taken if he were to attempt to raise water from
a lower level, we may prove as follows. Let there
be a siphon with its inner leg longer and narrow,
and the outer much less in length but broader so
as to contain more water than the longer leg.
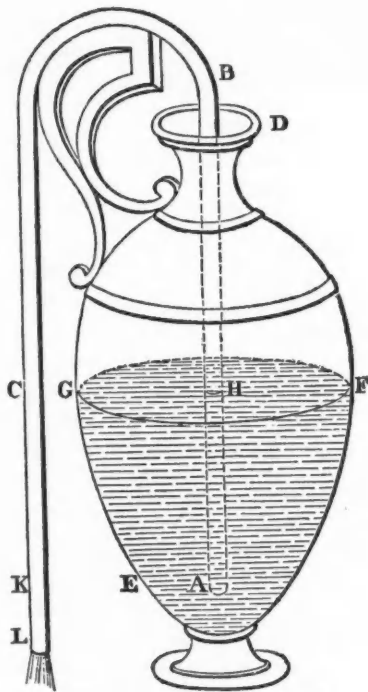
[9] See No. 60 in this series of "Reproductions," Am. J.
Phys. **22,** 175 (1954).
[10] Translation by J. G. Greenwood in *The Pneumatics of
Hero of Alexandria,* edited by Bennet Woodcroft (Taylor,
Walton and Maberly, London, 1851).

Then, having first filled the siphon with water, plunge the longer leg into a vessel of water or a well. Now, if we allow the water to flow, the outer leg, containing more than the inner, should draw the water out of the longer leg, which will at the same time draw up the water in the well; and the discharge having begun will exhaust all the water or continue forever, since the liquid without is more than that within. But this is not found to be the case; and therefore the alleged cause is not the true one. Let us then examine into the natural cause. The surface of every liquid body, when at rest, is spherical and concentric with that of the earth; and, if the liquid be not at rest, it moves until it attains such a surface. If then we take two vessels and pour water into each, and, after filling the siphon and closing its extremities with the fingers, insert one leg into one vessel plunging it beneath the water and the other into the other, all the water will be continuous, for each of the liquids in the vessels communicates with that in the siphon. If, then, the surfaces of the liquids in the vessels were at the same level before, they will both remain at rest when the siphon is plunged in. But if they were not, as soon as the water is continuous it must inevitably flow into the lower vessel through the channel of communication, until either all the water in both vessels stands at the same height, or one of the vessels is emptied. Suppose that the liquids stand at the same height; they will of course be at rest, so that the liquid in the siphon will also be at rest. If, then, the siphon be conceived to be intersected by a plane in the surface of the liquids in the vessels, even now the liquid in the siphon will be at rest, and, if raised without being inclined to either side, it will again be at rest, and that whether the siphon is of equal breadth throughout or one leg is much larger than the other. For the reason why the liquid remained at rest did not lie in this, but in the fact that the apertures of the siphon were at the same level. The question now arises why, when the siphon is raised, the water is not borne down by its own weight, having beneath it air which is lighter than itself. The answer is that a continuous void cannot exist; so that, if the water is to descend, we must first fill the upper part of the siphon, into which no air can possibly force its way. But if we pierce a hole in the upper part of the siphon, the water will immediately be rent in sunder the air having found a passage. Before the hole is bored, the liquid in the siphon, resting on the air beneath tends to drive it away, but the air having no means of escape does not allow the water to pass out: when however the air has obtained a passage through the hole, being unable to sustain the pressure of the water, it escapes. It is from the same cause that, by means of a siphon, we can suck wine upwards, though this is contrary to the nature of a liquid; for, when we have received into the body the air which was in the siphon, we become fuller than before, and a pressure is exerted on the air contiguous to us, and this in turn presses on the atmosphere at large, until a void has been produced at the surface of the wine, and then the wine undergoing pressure itself will pass into the exhausted space of the siphon; for there is no other place into which it can escape from the pressure. It is from this cause that its unnatural upward movement arises."

Chicago Section

At the annual spring meeting of the Chicago Section of AAPT, held on April 24, the following officers were elected for 1954–55. President: WILLIAM R. ANDERSON, *University of Illinois—Navy Pier*; Vice President: JAMES W. LUCAS, *Du Sable High School*; Secretary: RICHARD T. O'CONNOR, *W. M. Welch Scientific Company*.

The Stroboscopic Effect and the Barber-Pole Illusion

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(Received October 14, 1953)

Mathematically and experimentally this paper shows that the visual stroboscopic effect has as a limiting case the barber-pole illusion. The conclusion is reached with primary emphasis on physical stimulus rather than psychological, including physiological, response. The treatment is from the teaching standpoint. For readers unfamiliar with the stroboscopic effect its essentials are given in detail.

INTRODUCTION

THE barber-pole illusion rests, as O'Connell[1] pointed out, on mistaken identity, but he made no mention of the stroboscopic effect. His discussion was qualitative and from the broad standpoint of physical-psychological relationships.

It occurred to me that at least an analogy might be drawn between that illusion and the stroboscopic effect, to which I once gave some attention.[2] This effect also depends on mistaken identity. A subsequent comparison, to be described here, showed that the barber-pole illusion can be regarded as stroboscopic in character, being mathematically a limiting case.

This conclusion was reached in a quantitative physical manner, which means without the necessary presence of an observer directly viewing illusional motion. One could substitute for the observer a photocell in circuit with a galvanometer. Since the eye would not then participate, except incidentally to read the galvanometer, there would seem to be, in order to establish the main thesis here, no absolute necessity for considering the psychology of illusional motion directly observed visually. However, because in its fundamental character the stroboscopic effect has mathematically the barber-pole illusion as a limiting case, this point of view may be of interest in psychology and biophysics.

First, the barber-pole illusion will be described and a mathematical formula for the apparent velocity given. Then, for readers not acquainted with the principles of the stroboscopic effect, and to emphasize the subject as it might be used by a teacher, it will be presented in some simple, specific cases. This much familiarity is needed to understand how it becomes as a limiting case the barber-pole illusion.[3] Finally, the barber-pole illusion will be analyzed in terms of the stroboscopic effect.

THE BARBER-POLE ILLUSION

The oblique-striped design on a barber pole gives it the appearance of a screw. As the pole turns, usually clockwise seen from above, the stripes appear to rise continuously, just as the threads of a screw rotating about its own axis without motion along that axis would appear to do. A screw is essentially an inclined plane, the base here being horizontal, wrapped round a vertical cylinder. Unwrapping one turn from the pole gives a right triangle, Fig. 4(a). The base C equals the circumference of the cylinder, the altitude P the pitch of the screw. If B is the cylinder diameter, that of the barber pole, then $P = C \tan\theta = \pi B \tan\theta$. For a given pole, B and θ are constants.

The surface of the pole moves only horizontally, but the edge of the stripe seems to move vertically, and would still in effect do so with a photocell substituted for the eye, actuating an electromagnet. How fast does a stripe appear to move up the pole when it is rotating at frequency f? Obviously, for a single rotation the stripe—to be more specific, one edge of it—rises a distance P, so that the distance it moves in one sec is $Pf = V$, the apparent velocity of the stripe upward. If for the same rotational direction of the pole θ is negative, the illusional velocity will be downward, the hypotenuse of the triangle in Fig. 4(a) then sloping downward.

[1] See abstract, Am. J. Phys. **20**, 194 (1952).
[2] L. E. Dodd, Proc. Iowa Acad. Sci. **24**, 221–229 (1917).

[3] Another kind of barber-pole illusion, not pertinent here, is given by R. S. Woodworth in *Psychology, a Study of Mental Life* (Henry Holt and Company, 1921), p. 458.

Or, if the pole rotates counterclockwise as seen from above and θ is positive, V will be downward; if θ is negative, the illusional velocity will be upward.

In the first case, substituting, we have $V = (\pi B \tan\theta)f$. In terms of the actual linear speed v, tangentially in a horizontal plane, of a fixed point on the surface of the pole—suppose at the upper edge of a stripe, on the hypotenuse of the inclined plane—we have $v = \pi B f$, so that $V = v \tan\theta$.

THE STROBOSCOPIC EFFECT

The general stroboscopic effect[4] has basic characteristics common to particular manifestations of it. In laboratory and shop the visual stroboscopic effect is used with objects moving too fast for adequate observation, particularly for measuring high speeds and inspecting detailed parts that, because of their velocity, are invisible as such.

In the visual stroboscopic illusion two physically existent elements are essential: first, an object such as a surface bearing a series of suitably spaced similar details, that has a velocity v and may be called the stroboscopic or object screen, the similar details being referred to here as stroboscopic figures; and second, flash-illumination of this object screen intermittently, and particularly periodically at suitable intervals with frequency ν, resulting in the stroboscopic effect. This effect consists of a visible stroboscopic "image," which may or may not be in motion.[5]

[4] A broader term appears desirable, since "stroboscopic" implies the visual sense, while there is a similar effect tactually. Professor E. G. Boring, in *Introduction to Psychology* (John Wiley and Sons, Inc., New York, 1939), p. 493, mentions "apparent tactual movement." An experimental demonstration of this was described by me [see Proc. Iowa Acad. Sci. 28, 113–4 (1921)]. Both with the tactual and the auditory senses illusional velocity is possible. Complete discussion of these three manifestations of the "stroboscopic" effect must include psychology. In this paper the visual effect only is discussed, with emphasis on physical stimulus rather than physiological-psychological response.

[5] For a concise historical summary of devices for demonstrating the effect, see E. G. Boring, *Sensation and Perception in Experimental Psychology* (Appleton-Century-Crofts, Inc., New York, 1942), pp. 588–592. The similarity of Plateau's spiral (Fig. 95, p. 592) to the barber pole is obvious, but not mentioned there. In the spiral, the figure as a whole appears to contract or expand, due it would seem to the apparent linear motion of any segment perpendicular to its actual motion (barber-pole illusion). With the pole itself there can also be the illusion of rotation (see later in this paper).

If the screen velocity v is constant and the figures equally spaced, they will have a definite frequency ν_f of transit past a fixed reference line, in this paper taken perpendicular to the straight row of figures. The relation between this frequency and that of the illumination ν determines the illusional or stroboscopic velocity V in the effect seen. If ν is too low, then regardless of the value of ν_f, visual response will not continue through the dark interval between flashes, and the image will be intermittent, but will be stroboscopic as we define the term, involving mistaken identity. (Here we except the case where ν_f is so low that the same figure is seen at successive flashes in somewhat changed positions.)

STROBOSCOPIC-SCREEN VELOCITY ALONG DIRECTIONAL LINE OF STRAIGHT ROW OF STROBOSCOPIC FIGURES, FIG. 1

In Figs. 1 and 2 the stroboscopic figures are identical spots or dots, equally spaced a distance D_0 and arranged in a straight line on a plane surface as object screen. The screen velocity v is constant and in line with the row of dots. If illumination is continuous there will be no illusion in what the observer sees, whether or not the screen moves. That is, the individual dots maintain their identities in his mind, unless v is high enough to make the rapidly moving row appear blurred.

FIG. 1. Stroboscopic effect with single straight row of equally spaced dots as stroboscopic figures. $\nu =$ illumination frequency; $\nu_f =$ dot frequency; $v =$ velocity of object screen carrying the dots; $V =$ velocity of the stroboscopic image.

Case I

With a 1-to-1 ratio of the two frequencies, $\nu_f/\nu = 1$, we have the stroboscopic velocity $V = 0$.[6] The eye "sees" a row of apparently stationary dots, whether or not, as mentioned, the illumination frequency ν is high enough for visual response to mask the intermittency of the light. Each dot seen is a stroboscopic image, interpreted as a single dot on the object screen. Actually, this image is formed of a succession of different dots each in turn illuminated at the instant it is at the fixed location of the image.

Let the row of dots, $a, b, c, \cdots$, move along their directional line with constant velocity v, a flash occurring at $t = 0$, the instant when they have the position shown. Let the time unit be Δt, the constant time interval between flashes. Then $\Delta t = 1/\nu_f = 1/\nu$. At the end of the first time interval, when the next flash occurs, dot b is at the previous position of dot a, dot c has replaced dot b, and so on. At the end of the second Δt, dot c has moved up to the initial position of a, dot d to that of b, etc. To the observer, however, dot a remains stationary, and likewise dots $b, c, \cdots$, and there will be seen an apparently stationary row of dots, that is, a row of stroboscopic images. For this case, with a definite frequency ν of the flashes, and with the condition $\nu_f = \nu$, we have $v \equiv v_0 = \nu_f D_0$, and $V = 0$.

This particular case has had much practical application for calibration of frequencies, where comparison is made with a standard frequency, as that of a tuning fork. Such technique has advanced, so that today the tuning fork, for example, can be controlled by the natural oscillation of a quartz crystal, which in turn can be controlled by vibration in the ammonia molecule.

Case II

If now the object screen is speeded up somewhat so that $v > v_0$ and constant, the flash frequency ν remaining the same, we have $\nu_f > \nu$, while $\Delta t = 1/\nu$, as before. The obvious effect is that at $t = 1$ unit, dot b is slightly beyond, that is to the left of, the initial position of a, dot c correspondingly at the left of b's initial position,

[6] In general, if the frequency ratio is that of any two different numbers prime to each other, the stroboscopic condition $V = 0$ will be present. This paper is not concerned with such cases.

and so on. At this and successive flashes, the situations are as shown. The row of dots as a whole appears to be moving relatively slowly to the left. This is the illusional stroboscopic velocity V and its direction is along the row of dots. Indication of V as positive in the diagram means it has the same direction as v, which being to the left here will fit in with the later discussion of the barber pole.

Case III

Here the object screen has been slowed down a little, so that $v < v_0$, whence $\nu_f < \nu$, since ν remains the same. The stroboscopic velocity is toward the right, specifically again in line with the row of dots.

SCREEN VELOCITY NOT IN DIRECTION OF ROW OF STROBOSCOPIC FIGURES, FIG. 2

How is it possible for stroboscopic velocity to have a component at right angles to the motion of the stroboscopic screen, or even to be entirely at right angles to that motion? This is clear from Fig. 2, where the screen velocity v is not in line with the straight row of stroboscopic figures, kept horizontal in the drawings. The v vector is taken first in the third quadrant, downward to the left. Here ν_f continues to represent the frequency with which the dots pass a fixed vertical index line held perpendicular to the row.

Case I. $\nu_f > \nu$

The stroboscopic velocity V is downward toward the left, the same as the constant screen velocity v, but is not in the direction of v.

Case II. $\nu_f = \nu$

V is now at right angles to the row of dots, but not to v.

Case III. $\nu_f < \nu$

The V vector is in the 4th quadrant, downward to the right, and the ratio ν_f/ν is of such a value that V *is perpendicular to* v. This case is applicable to the barber-pole illusion.

Case IV

The ratio ν_f/ν is considerably less than in Case III. The V vector, stroboscopic velocity, points still farther to the right.
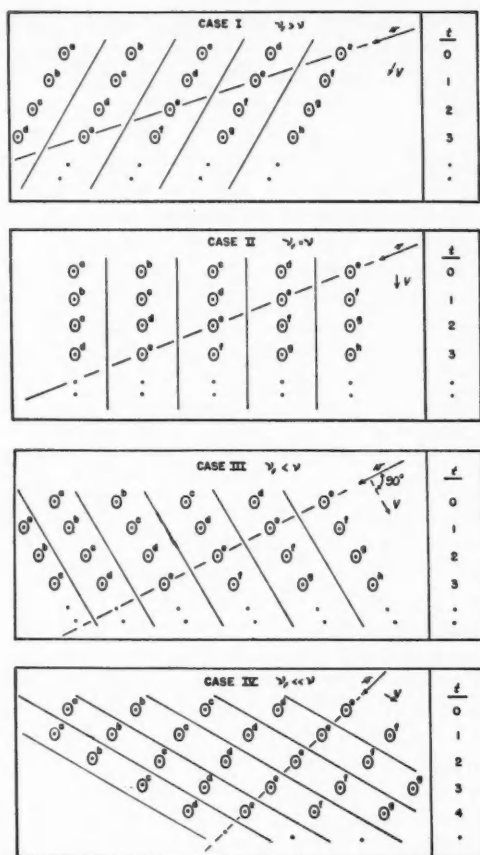
FIG. 2. Similar to Fig. 1, but with screen velocity v oblique to directional line of dots. Case I, V at acute angle with v; Case II, V at right angles to line of dots; Case III, V at right angles to v; Case IV, V at obtuse angle with v.

It is thus seen from the four cases of Fig. 2 that V may be at any angle with v. This is amply illustrated on the motion picture screen, where one can see an automobile traveling horizontally, a man climbing or descending from a ladder, or a bird flying, say, upward to the right.

LANTERN DEMONSTRATION OF STROBOSCOPIC EFFECT

Lantern projection of the stroboscopic illusion is easy. A vertical-type lantern where slides are laid flat on a horizontal stage may be preferred. Perforated cards simulate the intermittent illumination of similar spots serving as stroboscopic figures. For any one demonstration, two

only of these cards, superimposed on the lantern stage, are needed. One of them, called the trace card, is next to the stage and usually held stationary. The other, the object card, carries perforations as stroboscopic figures. The cards must have relative horizontal motion, one sliding over the other. This physical motion is reversed, of course, on the lantern screen. However, the projected stroboscopic motion is not in general this horizontal motion, and it can have any direction in the plane of the screen, as will be shown.

Figure 3 shows three trace cards, A, B, C, each having a parallel-sided slit-aperture. The trace card determines the path of the illusional stroboscopic motion. Seven cards, M to S, act as object screens, the first three carrying equally spaced similar apertures as stroboscopic figures.

When an aperture in the object card (for
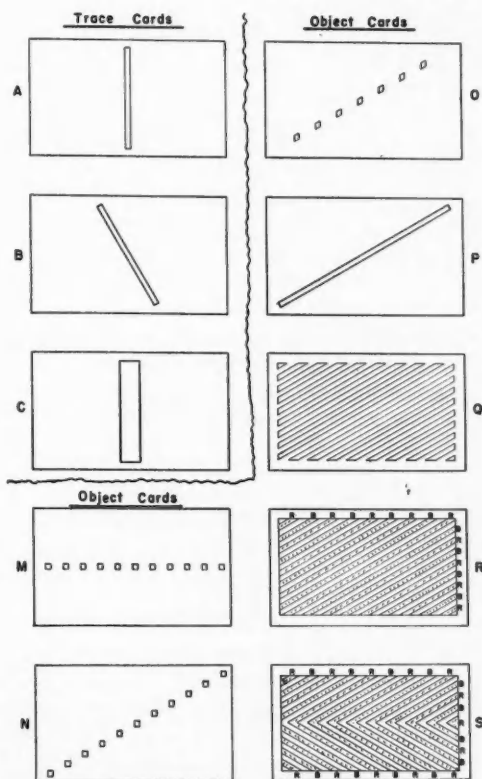
FIG. 3. Chart of trace cards and object cards used in lantern demonstration.

cards M, N, O) coincides with the opening in the trace card (either A or B), light passes through both cards to the lantern screen, where a real image of the hole appears on a dark background. This image appears only briefly, its flash being succeeded a moment later by the similar flash of the image of the next aperture. Thus, flash illumination of stroboscopic figures is simulated. If the relative motion of the two cards is fairly fast, the eye interprets what it sees on the lantern screen as the repeated image of the first aperture in the object card, and so on for the succeeding apertures in the series. Giving further details we shall describe a series of demonstrations leading up to that of the barberpole illusion as a limiting case.

PARTICULAR DEMONSTRATIONS USING PERFORATED CARDS

The cards shown in Fig. 3 were prepared, except R and S, by perforating ordinary 3×5 file cards with a razor blade. For easier handling in the lantern, the object cards, except R and S where the colored fields are 6 inches long, can be extended lengthwise by attaching each to similar cards end to end with Scotch tape, then reducing the total length as desired. Also, a convenient folding holder for all the object cards is made from two similar strips of lantern-slide glass $3\frac{3}{4} \times 10$ in., hinged along one edge with tape.

1. Zero and Plus and Minus Finite Values of the Stroboscopic Velocity V, Parallel to Object-Screen Velocity v and to the Stroboscopic Figures

We start with trace card A, Fig. 3, where the opening is vertical. As object card use M, with its horizontal row of equally spaced holes $\frac{1}{8}$ in. square and $\frac{7}{16}$ in. apart between centers, the edge of each square being $\frac{1}{8}$ in. and equaling the width of the slit. When this card is drawn horizontally at a uniform rate over the trace card, a succession of flashes, that is, real images of the square holes, appears in the same place on the lantern screen. The observer readily interprets what he sees as the image of the same hole appearing periodically at the same place. This is the stroboscopic effect where $V=0$. Card M does not need to move very rapidly

before the observer neglects horizontal motion of the vertical edges of the aperture images, as they come out of and re-enter eclipse. In motion picture projectors, fast acting shutters are used. If the card moves fairly fast, visual persistence masks the intermittency so that one sees a single steady image and the illusion is complete.

In this stationary stroboscopic state, the frequencies of illumination and of passage of the holes or stroboscopic figures v_f past a fixed point on lantern stage or screen are the same. By also moving trace card A slowly relative to the speed of object card M, one changes the ratio of the two frequencies from unity to a value either greater, if A moves in the same direction as M, or less, if A moves in the direction opposite to that of M. We now have on the screen what appears to be the same spot of light moving relatively slowly in the horizontal direction, to either right or left depending on the ratio v_f/v.

A current method for demonstrating the effect uses a rotary disk having concentric circles of equally spaced dots as stroboscopic figures, the total number of these increasing from one circle to the next. The disk is run at a suitable constant speed and periodically illuminated, as with a strobotac, or strobolux, or a stroboconn.[7] Because the duration of each flash is short, of the order of 10^{-6} sec, and the intensity of illumination high, the stroboscopic images are sharper than with the old-time manometric flame actuated by sound waves, and other older devices. If for a given circular row $v_f = v$, that row will appear stationary, while circles on one side of it will seem to rotate slowly and with constant speed clockwise, this speed increasing from circle to circle, while on the other side the effect will be similar but oppositely directed.

2. Stroboscopic Velocity Oblique to Screen Velocity

As seen especially in motion pictures, which are always stroboscopic images, and in television, the velocity V may be zero or have different

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[7] The strobotac and strobolux are manufactured by Edgerton, Germeshausen, and Grier, Inc., Boston, Massachusetts, and the stroboconn by C. G. Conn, Ltd., Elkhart, Indiana. Practical uses are to inspect rapidly moving mechanical parts seen as stationary or in slow motion, and to test with high precision the pitch of a sounded tone, also to determine mechanical frequencies in general.

values in any direction relative to that of the film. Successive photographs of the same object detail supply the stroboscopic figures. In motion pictures, apparent velocity is obtained by relative displacements of the same detail on successive frames of the film, with a fixed speed through the projector.

To demonstrate V in an oblique direction, use trace card B, in which a straight slit is cut obliquely, with object card N, where the similar holes are in an oblique rather than horizontal row, it being at about 30° with the horizontal. Keep B stationary. Obviously the direction of the apparent velocity is that of the oblique opening. If B also is moved slowly relative to N in the horizontal direction, a component can be added, so that V now has an oblique direction different from that of the opening in B and the row of apertures in N.

3. V at Right Angles to v

Since, as mentioned, the barber pole is essentially a right-handed screw with rotation usually clockwise as seen from above, the apparent motion of the stripes is upward where θ is positive, Fig. 4(a), and thus at right angles to the actual motion of any point on the pole's surface. Use card A with N. With A stationary move N so that its image, simulating the surface of the pole, moves from right to left on the screen. The "same" spot on the screen now rises vertically upward.

To approach nearer the barber-pole effect, use cards A and O, the latter a simple modification of N. The equal holes, instead of being square, have diamond shape, with the two sides vertical and of the same length as before, and separated as before, the top and bottom edges being cut parallel to the line of centers. This line is at 30° with the horizontal. Again on the lantern screen the stroboscopic velocity is upward, but the stroboscopic image now has the new shape of the holes, suggesting an inclined plane.

4. The Barber-Pole Demonstration

Next, substitute for card O object card P. This has a single straight slit at 30° as in N and O. With the same trace card A, the visible effect on the screen now is that of a single diamond-shaped spot of light rising uninterruptedly. Card P differs from card O only in that the spacing between the holes in O has been reduced until they are in contact. We thus have clearly a limiting case of stroboscopic effect. The similarity to the rising stripe on a barber pole is more striking if we substitute for card A, with its narrower slit, card C with a similarly vertical but considerably wider opening.

For a more complete barber-pole demonstration, use object card Q, its only difference from P being that it has a number of equally wide parallel slits with the same obliquity as before. The width of cardboard remaining between slits is the same as that of the slits, which is that of the single slit in card P.

More realistic still, object card R is similar to Q, but there are no opaque spaces. Strips of colored cellophane are mounted between colorless sheets, in the sequence red-white-blue-white-red-, and so on. The white strips seen on the lantern screen are vacant spaces between the colored ones. All strips have the same width, preferably that of the slits in cards P and Q. To preserve the sequence, object card R is used at first with trace card A, but the effect is more realistic using C with its wider aperture.

In some barber poles, there are two fields. On the upper half the stripes travel upward, on the lower half downward. To demonstrate this we have two sets of parallel stripes, one inclined at angle $\theta = 30°$, the other at minus θ, card S. Stripes of the same color in the two sets should join neatly, coming to a nice point.

The stripes in this demonstration have straight edges. In watching a barber pole one sees the edges somewhat S-shaped, in perspective. Actually, they are the visible segments of a helix with θ somewhat variable, but the observer of the demonstration is not likely to note the difference. The basically stroboscopic character of the illusional motion is present in both cases.

ILLUSION OF ROTATION

With cards C and R, added to the illusion of the stripes rising is the three-dimensional one that the pole is rotating. This appears to be due both to the rising stripes and the horizontal movement of visible details on the card, suggesting such on the surface of an actual revolving pole.

DETAILED ANALYSIS OF THE BARBER-POLE EFFECT

Briefly, the discussion above preliminary to describing the lantern demonstrations, especially the discussion of Fig. 2, Case III, will be applied analytically to the barber pole. For the time being suppose the pole is in darkness except when flash illuminated at regular intervals. The instantaneous positions of the edge of a stripe at three successive flashes are shown in Fig. 4(b). Let a, a', a'' $\cdots$ and b, b', b'', $\cdots$ and so on, be points as stroboscopic figures, equally spaced along this edge, with separation distance D_0. The stripe has the actual velocity v to the left, its displacement between flashes being $D_0 \cos\theta$. Although these points move only horizontally, because of mistaken identity the eye interprets them as moving vertically upward, so that points a' and a'' are mistaken successively for point a, and so on for the other points.

The algebraic relations given in Fig. 4 are obvious. Since $V = (\nu D_0) \sin\theta$, and since also $V = v \tan\theta = $ a constant, the product νD_0 is constant. If we now imagine D_0 reduced, and the illumination frequency ν correspondingly increased, the value of V is not changed. Next, let $D_0 \sim 0$, so that the actual edge of the stripe is thought of as made up of an indefinitely large number of equally spaced points, while $\nu \sim \infty$,

the periodic flash-illumination becoming continuous illumination. We now have the stroboscopic situation in the barber pole. Practically, the limit of D_0 does not have to be zero, because of the finite resolving power of the eye. Here is meant the optical resolving power in the usual sense, a *space* consideration, although the persistence of the visual impression from a flash source, which is a *time* consideration, is involved in viewing the stroboscopic effect.

To put it another way, the visible line-segment ab, Fig. 4(b), may be regarded as stroboscopic figure. At the first flash it is at the lower position shown; at the second flash, in the middle position; and so on. The eye interprets it as the *same* segment. Actually, it is somewhat different in each successive position. At the second flash, a piece $aa' = D_0$ has dropped off the lower end of ab and another piece bb' of equal length has been added at the upper end. The different segments $a'b'$, $a''b''$ are all identified as segment ab which would then appear to be rising. There is *a continually changing identity of the visible segment as the pole rotates*. A rough illustration is that of a not-too-hypothetical motorcycle that through the years undergoes changes. First, the rear wheel has to be replaced, then the front wheel, then the motor and finally the frame. But there is no intention here of considering psychological or philosophical aspects of the problem of identity.
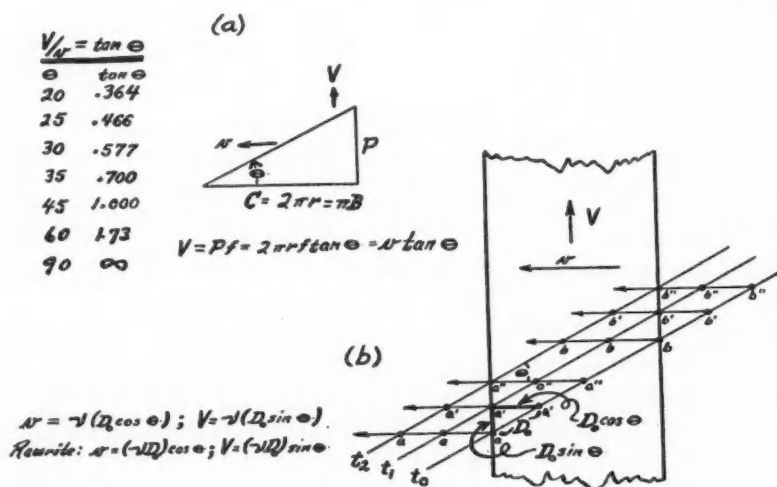
FIG. 4. The barber-pole illusion as stroboscopic effect. (a) Upper left: relation between (1) the illusional velocity V and (2) the diameter $B = C/\pi$, the pitch P, the inclination θ of the stripes and the rotational frequency f of the pole. (b) Lower right: analysis of the barber pole as a limiting case of the stroboscopic effect, where the dot-separation distance $D_0 \sim 0$ (continuous edge of a stripe), and the illumination frequency $\nu \sim \infty$ (continuous illumination).

IS THERE VERTICAL MOTION IN THE PHYSICAL SENSE?

Contrary to the observer's impression, no vertical motion exists in the barber pole. However, there exists physical motion vertically having to do with the observer's eyes.[8]

If his line of vision remains fixed, toward the central part of the pole, let us suppose, with the eyeballs stationary, there is on his retina a real image of stroboscopic character. This, according to our view above, is mathematically an infinitely, or quasi infinitely, rapid succession of similar real images at continually changing points on the retina, but interpreted as one image.[9] Each image is physically real and so are the displacements on the retina and retinal stimulation. The observer is not deluded in these respects.

Or, if his eyes follow upward a stripe on the pole, then the eyeballs are rotating about a horizontal axis. The anterior point of the cornea moves upward while the stroboscopic image of the edge of the stripe retains its fixed position on the retina. As in the preceding case, the stimulation of the retinal nerves is just as real physically as if the image there were of a single physical object moving in the direction of the illusional motion. Probably both effects are varyingly present with a given observer. He is by no means totally deluded. But he is dealing with the stroboscopic effect where the illusional motion is at right angles to the actual motion of the barber pole.

ANALOGOUS EXAMPLES

Are there other examples of mistaken identity that might be regarded as belonging to the stroboscopic effect? Such classification is conceivable with a candle flame. Unless, as a scientist, he thinks of the flame in terms of vibrations in atoms, the observer of it regards it as the same object, having continued existence. But mistaken identity is present, for molecules flow into the flame and out of it, in a stream of which the flame may be regarded as a luminous part.

At first, one might not see clearly here the presence of the two frequencies characteristic of the stroboscopic effect. But there is a striking and even a close similarity to the flow of a sequence of photographic images of a detail that appears stationary on the motion picture screen, into and out of the stroboscopic image that the audience sees there. In fact, any visible source of light can be considered similarly, if analysis is carried to the intramolecular sources of the myriad flashes of light.

A second example is found in meteorology. The horizontal base of a cumulus cloud appears stationary. Actually, the cloud grows downward in the air stream at the same rate as the air stream carries the cloud upward.

This example suggests still another, often seen. In a stream of water rapidly descending over a rough bed, there may be present a succession of rather regularly spaced undulations, stationary with respect to the ground. This is actually a train of waves moving upstream through the water at the same rate the water flows with respect to the ground.

ADVANTAGES OF THE PRESENT DEMONSTRATION METHOD

(1) The present demonstration is easily shown to a large group, while the usual figured disk for demonstrating the stroboscopic effect is limited in size, so that clear viewing needs a shorter distance. (2) For instruction, the two essential frequencies are low enough to permit the periodicities to be readily appreciated. Briefly, the stroboscopic effect can be "taken apart" experimentally to "see how it works." (3) The equipment accessory to the lantern is simple, easily prepared, and inexpensive.

Finally, I wish to thank Lewis Humason for preparation of the colored object cards.

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[8] See E. G. Boring, "Triple set of cues for the perception of visual movement," in *Introduction to Psychology* (John Wiley and Sons, Inc., New York, 1939), p. 487.

[9] Since discreteness characterizes the retinal nerve endings, the mathematical limit does not need to be reached. But we shall not discuss here visual acuity or resolving power.

On Planck's Quantum of Action

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(Received January 26, 1954)

It is shown by means of dimensional analysis that both the Stefan-Boltzmann law and Wien's displacement law imply the existence of Planck's constant of action or of an equivalent constant.

1. INTRODUCTION

IN courses on heat radiation the students are usually informed about the early gropings of the theory and told how the formidable difficulties were finally surmounted by Planck's quantum hypothesis. Almost regularly the reaction of the listeners is expressed in the following questions. How could Planck be sure that such a revolutionary hypothesis as that of the existence of the quantum of action was necessary? Why was he convinced that a less drastic assumption could not lead to a reasonable theory of heat radiation? The historical fact is that the same questions were asked by most of the contemporary physicists and that Planck himself was not entirely convinced but supported his own hypothesis only tentatively and halfheartedly. The general acceptance of the quantum idea was due to its great successes in fields other than the theory of radiation, most strikingly, in those of the photoelectric effect and the theory of the atom.

Nevertheless, there exists a simple and satisfactory way to answer the students' questions without wandering into extraneous fields. It is the way of subjecting the conceptions of the theory of heat radiation to a *dimensional analysis*. This approach was used in the writer's lectures for almost thirty years and proved to be of considerable instructive and pedagogic value, always meeting with a favorable response from the listeners.

2. DIMENSIONAL ANALYSIS

We shall need the method of dimensional analysis in a form somewhat different from that in which it is found in most textbooks. It shall be taken as known that every physical observable has a definite dimension in each of a set of basic quantities. To fix our ideas we shall use the example of the very observables which we wish to discuss later on, namely, those characterizing heat radiation. In this case, the basic quantities in terms of which all observables can be expressed are mass m, length l, time t, and temperature T, so that an observable X satisfies the dimensional formula

$$[X] = [m^\mu l^\lambda t^\tau T^\sigma],$$

μ, λ, τ, and σ being numbers. For instance, the total *energy density* of a radiation field is

$$[u] = [ml^{-1}t^{-2}], \tag{1}$$

and the *monochromatic energy density* (i.e., the density corresponding to the frequencies between ν and $\nu+d\nu$ divided by the interval $d\nu$) is dimensionally expressed by

$$[u_\nu] = [ml^{-1}t^{-1}]. \tag{2}$$

The frequency itself obviously has the dimension

$$[\nu] = [t^{-1}].$$

Sometimes it is known from general considerations that the observable X is a function of certain other observables, for instance, $Y_1, Y_2, Y_3,$

$$X = f(Y_1, Y_2, Y_3). \tag{3}$$

This is the case when the dimensional expressions become useful. As an example, let us consider an evacuated cavity with solid walls of the temperature T. It was shown by Kirchhoff that the density of radiation inside such a cavity is a function of the temperature only

$$u = f(T),$$

while the monochromatic density is a function of temperature and frequency only

$$u_\nu = F(T,\nu).$$

Both functions are *universal* in that they do

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not depend on the nature of the cavity walls with which the radiation is in contact. At first sight one may be inclined to write a dimensional relation in the form $[u_\nu] = [T^\alpha \nu^\beta]$. However, this would be a mistake for the following reason: although the universal functions f and F depend on no other variables than T and ν, they may be expressed in terms of some dimensional universal constants $c_1, c_2, c_3, \cdots$ so that the equation could be given the form

$$u = f(T, c_1, c_2, c_3, \cdots), \qquad (4)$$

$$u_\nu = F(T, \nu, c_1, c_2, c_3, \cdots), \qquad (5)$$

or generally

$$X = \Phi(Y_1, Y_2, Y_3, \cdots c_1, c_2, c_3, \cdots). \qquad (6)$$

The appropriate dimensional equation is, therefore,[1]

$$[u_\nu] = [T^\alpha \nu^\beta c_1^{\gamma_1} c_2^{\gamma_2} \cdots],$$

or generally

$$[X] = [Y_1^{\eta_1} Y_2^{\eta_2} \cdots c_1^{\gamma_1} c_2^{\gamma_2} \cdots]. \qquad (7)$$

Inasmuch as the universal constants themselves satisfy dimensional relations of the type

$$[c_j] = [m^{a_j} l^{b_j} t^{d_j} T^{f_j}], \qquad (8)$$

both sides of Eq. (7) can be represented as products of powers of the four basic quantities. Obviously, the power of each quantity must be the same on both sides, and this gives four equations which the exponents $\eta_1, \eta_2, \cdots \gamma_1, \gamma_2, \cdots$ of Eq. (7) must satisfy. On the other hand, let the number of arguments in Eq. (6) be n, so that Eq. (7) contains n exponents which are the unknown quantities in the four equations. One of the three following cases will then occur:

(1) $n = 4$. The number of arguments is equal to the number of basic quantities. The four equations have then, in general, a unique solution and the form of the function in Eq. (6) is completely determined.

(2) $n < 4$. When the equations are more numerous than the variables they have, in general, no solution. As every correctly stated question of physics must have an answer, this is an indication that the Eq. (6) giving the relation be-

tween observables and universal functions was chosen incorrectly and must be replaced by another relation.

(3) $n > 4$. The exponents η, γ and the form of the function are then not uniquely determined. The treatment of this case will become clear through the discussion of a special example in Sec. 4. We shall state here only the general conclusions:

(a) There exist $n - 4$ nondimensional (i.e., pure numbers) aggregates of the observables $Y_1, Y_2, \cdots$ and of the universal constants $c_1, c_2, \cdots$.

(b) The expression (6) of X contains an arbitrary function of these aggregates.

Although the example of Sec. 4 refers to the case $n - 4 = 1$, the generalization to any $n - 4$ is obvious.

3. CLASSICAL THEORY

In returning to the theory of heat radiation we are faced with the question: what are the universal constants in Eqs. (4) and (5)? It is to be expected that the velocity of light c and Boltzmann's constant k play a role in the theory of radiation. Indeed, c characterizes the propagation of radiation in vacuum and k is distinctive of all statistical processes and always accompanies the temperature which is itself a statistical quantity. Their dimensional expressions are as follows:

$$[c] = [lt^{-1}], \quad [k] = [ml^2 t^{-2} T^{-1}].$$

In the classical theory the constants c and k are the only ones that are pertinent to the problem. It is true that the classical theory recognizes other universal constants, for instance, the elementary electric charge e. However, these are constants characterizing matter and not radiation. According to Kirchhoff's law the equilibrium radiation in a cavity is the same independently of the nature of the walls or independently of the nature of the matter with which it is in equilibrium. This applies not only to actually existing matter but also to matter that we construct theoretically, provided that the construction is done according to the laws of classical physics. Inasmuch as we can imagine oscillating charged particles emitting radiation

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[1] The role of the universal constants in dimensional analysis was clarified especially by P. Straneo, Atti accad. nazl. Lincei 26, 271, 289 (1917).

with charges other than e, the elementary charge cannot be a characteristic of the radiation. The same applies to other universal constants describing matter. The more detailed investigations of the equilibrium of matter and radiation, due to Rayleigh and Planck, completely bear out this conclusion. Hence, the Eqs. (4) and (5) are reduced to the form

$$u = f(c,k,T), \tag{9}$$

$$u_\nu = F(c,k,T,\nu). \tag{10}$$

We begin with the monochromatic density

$$[u_\nu] = [T^\alpha \nu^\beta c^\gamma k^\delta],$$

substituting the expressions of u_ν and of the arguments in terms of the basic quantities, as expressed by the Eqs. (2), (3), and (8), and comparing the exponents we obtain the equations

$$\delta = 1, \quad \gamma + 2\delta = -1, \quad \beta + \gamma + 2\delta = 1, \quad \alpha - \delta = 0.$$

The solution is unique, namely

$$\alpha = 1, \quad \beta = 2, \quad \gamma = -3, \quad \delta = 1,$$

whence

$$u_\nu = A k c^{-3} \nu^2 T, \tag{11}$$

where A is a purely numerical constant. Equation (11) is known as Rayleigh's law of black radiation. Our treatment shows that it is the only law compatible with classical theory.

Turning to the total energy density, we notice that the expression (9) contains only three arguments, one less than the number of equations that must be satisfied. It is, therefore, *a priori* doubtful that it can be satisfied. Letting

$$[u] = [T^\alpha c^\gamma k^\delta],$$

we arrive from Eqs. (1) and (8) at the four conditions

$$\delta = 1, \quad \gamma + 2\delta = -1, \quad \gamma + 2\delta = 2, \quad \alpha - \delta = 0.$$

These conditions are indeed incompatible and admit of no solution. Thus the classical theory is incapable of giving an expression for the total energy density of radiation. This result is in keeping with the structure of Rayleigh's law (11). It was pointed out by Rayleigh himself that the integral

$$u = \int_0^\infty u_\nu d\nu$$

is divergent: if Rayleigh's law were true there would be no reasonable law for the total energy density.

4. QUANTUM THEORY

The inescapable conclusion is the same which was drawn by Planck: *the classical theory is inadequate.* In deciding in what way the classical theory must be changed, we must bear in mind the following considerations. In the first place, the form of the Eqs. (4) and (5) is based on thermodynamics and must be retained. In the second place, the universal constants c and k are clearly relevant to the observables of heat radiation. The only way to avoid the difficulties of the preceding section is, therefore, to assume that the observables u and u_ν depend, *in addition to c and k*, on a further universal constant which we shall designate by g.

$$u = f(c,k,g,T), \tag{12}$$

$$u_\nu = F(c,k,g,T,\nu). \tag{13}$$

It would be easy enough to show that all difficulties are resolved when g is identified with Planck's constant of action h. We shall, however, follow a different course which is more instructive because it will show that h (or a constant equivalent to h in a sense explained below) is the only one that has this property. We shall use an additional piece of information offered us by thermodynamics, namely, the Stefan-Boltzmann law which states that $u \propto T^4$. Taking this into account, the dimensional relation corresponding to Eq. (12) has the form

$$[u] = [c^a k^b g^d T^4]. \tag{14}$$

Our intention is to find the dimension of g, but it is easy to see that g is not unique. Indeed, if g is universal, then any power of g can be also regarded as universal and hence the exponent d in our equation is arbitrary being changed by a new definition of g. But this is not all, if we multiply the universal constant g by any power of c or by any power of k we get again universal constants. In this sense we can say that $g' = g^\lambda c^\mu k^\epsilon$ is a universal constant *equivalent to g* (λ, μ, ϵ being numbers). The upshot of this situation is that all three exponents a, b, d of Eq. (14) are arbitrary and can be chosen at our

convenience. We shall choose them in such a way as to obtain for g a particularly simple dimensional expression, namely, we let $a = -6$, $b = 4$, $d = -3$. Hence, we find

$$[g] = [u^{-1}c^{-6}k^4T^4]^{\frac{1}{2}},$$

or substituting formulas (1) and (8)

$$[g] = [ml]. \tag{15}$$

There is no loss of generality in this special choice: if we replace the g so defined by any universal constant g' equivalent to it, the Eqs. (12) and (14) will change only in form but not in essence.

Let us apply our result to the determination of the monochromatic intensity

$$[u_\nu] = [T^\alpha \nu^\beta c^\gamma k^\delta g^\epsilon].$$

Substituting the expressions (2), (3), (8), and (15) and comparing the coefficients we obtain the four conditions

$$\delta + \epsilon = 1, \quad \gamma + 2\delta + \epsilon = -1, \quad \beta + \gamma + 2\delta = 1, \quad \alpha - \delta = 0.$$

As there are five variables they cannot be determined uniquely but only in terms of an indeterminate parameter τ, for instance,

$$\alpha = \delta = -\tau, \quad \beta = 3+\tau, \quad \gamma = -2+\tau, \quad \epsilon = 1+\tau,$$

whence as special expression for the monochromatic density

$$u_\nu = A_\tau g c^{-2} \nu^3 (gc\nu/kT)^\tau, \tag{16}$$

again A_τ is a purely numerical constant. The right side of Eq. (16) has the same dimension for all values of τ; this implies that $gc\nu/kT$ is a *dimensionless aggregate* as, indeed, can be easily

checked by means of the formulas (3), (8), and (15). Furthermore, it is obvious that the most general expression for u_ν, preserving its dimensional requirements is obtained when all the special expressions of the type (16) are added,

$$u_\nu = gc^{-2}\nu^3 \sum_\tau A_\tau (gc\nu/kT)^\tau,$$

where A_τ are arbitrary coefficients. However, it is known from mathematics that the sum represents an *arbitrary function* of the aggregate $(gc\nu/kT)$ so that we arrive at Wien's law of displacement in the form

$$u_\nu = gc^{-2}\nu^3 f(gc\nu/kT).$$

The form becomes slightly more familiar if we introduce instead of g a universal constant h equivalent to it, in the sense explained above, namely: $h = gc$, whose dimension is obviously

$$[h] = [ml^2t^{-1}],$$

so that h can be identified with Planck's constant of action. Thus

$$u_\nu = hc^{-3}\nu^3 f(h\nu/kT). \tag{17}$$

This treatment makes it clear that Planck's constant of action is a prerequisite of both the Stefan-Boltzmann law and of Wien's displacement law.

These results should not be taken as a disparagement of Planck's great achievement. To obtain his law of radiation the mere knowledge of the existence of the constant of action was altogether insufficient and a deep insight into its physical nature was necessary as was provided by Planck's hypothesis of discrete energy states.

"*It is important to realize that a scientific theory is never susceptible to proof in the sense of logical demonstration. A theory is accepted as true when the known phenomena can be deduced from it in a simple and direct manner. . . . Almost any theory can be made to fit almost any phenomenon if a sufficiently ingenious supplementary hypothesis, invented for the purpose, is admitted. A theory is abandoned, not because it is disproved, but because it becomes too clumsy and complicated and is rejected to make way for a rival which fits in with the phenomena more simply. . . . The usefulness of a theory depends on its power to coordinate the facts of experience.*"
—ALEX WOOD, in *Thomas Young, Natural Philosopher* (Cambridge University Press, 1954).

NOTES AND DISCUSSION

Some Common Examples of Careless Statements in Elementary Physics Textbooks

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FROM time to time, articles appear that have as their purpose discussion of erroneous or careless statements in elementary textbooks. This is another paper with such a purpose. The following discussion is based upon an examination by the writer of twenty-five recent textbooks in elementary physics.

Centrifugal force has been the subject of a number of papers in the *American Journal of Physics* since 1935.[1-6] These papers have mainly consisted of arguments as to how centrifugal force should be defined. Some writers have maintained that it should be defined as a fictitious force acting on a body moving in a circle and that it be considered as the equilibrant of centripetal force. Others say that it should be defined as the reaction to centripetal force in accord with Newton's third law. The present paper is not concerned with this argument but is intended to point out that in a few books the authors are inconsistent in their discussions. At least three books less than ten years old define centrifugal force as the reaction to centripetal force, and then speak of the centrifugal force "on" a revolving body. One of these books is very clear with the definition as a reaction and then follows with three separate examples of centrifugal force acting "on" one thing or another.

Another subject that has had considerable discussion is the derivation of Bernoulli's equation.[7-13] Several papers dating back to 1925 have pointed out that it is not correct to assume the existence of potential energy per unit volume due to pressure in an incompressible fluid. Most recent textbooks treat this subject satisfactorily, but at least four books that have come out during the last ten years contain unsound statements about pressure energy.

My third example seems not to have been discussed in the literature. It involves the lack of distinction in many textbooks between the meanings of magnification and magnifying power in optics. Authors usually define magnification of lenses or mirrors as the ratio of image height to object height. Some then speak of the magnification of certain optical instruments where the meaning is quite different without specifically mentioning the distinction. Students who have seen magnification defined only as the ratio of image height to object height are shocked to be told that the magnification of an astronomical telescope is F/f. They might well wonder if a telescope having a value of 75, say, for the ratio of F to f, will form an image of the moon that is 75 times as big as the moon. Fortunately most texts avoid this confusion by defining a new term such as magnifying power, or angular magnification, to apply to the telescope and similar instruments.

My final example is in a slightly different category in that no incorrect, or careless, statements can be said to be involved. My reference is to Pascal's principle. In most books, it is stated about as follows:

> Pressure applied to any part of a fluid in a closed vessel is transmitted undiminished to all parts of the fluid.

A few books state the principle without reference to a closed vessel. An example is the following as stated in Eldridge's book.[14]

> When pressure is increased (or decreased) at one point in a fluid all points experience the same change.

Pascal referred to a closed vessel in his original rather verbose statement of the principle.[15] He began as follows:

> If a vessel full of water, otherwise completely closed, has two openings, one of which is one hundred times as large as the other; by putting in each of these a piston which fits exactly, etc.

With this statement, Pascal seems to have been the first to mention pressure transmission in closed vessels. However, as stated by Eldridge, the principle was recognized at least thirty years before Pascal was born. It was stated by Giovanni Battista Benedetti in 1585[15] and by Simon Stevin in 1586.[16] If the formulation as given by Pascal is taken as "Pascal's Principle," it is then stated as it should be in a majority of textbooks. On the other hand, Pascal's statement is a special case of the earlier formulation that makes no reference to closed vessels.

[1] C. F. Hagenow, 3, 190 (1935).
[2] V. F. Lenzen, 7, 66 (1939).
[3] R. Orin Cornett, 7, 347 (1939).
[4] Arthur Taber Jones, 11, 299 (1943).
[5] Arthur Taber Jones, 12, 233 (1944).
[6] Oswald Blackwood, 12, 233 (1944).
[7] E. H. Kennard, *Science*, 62, 243 (September 11, 1925).
[8] G. A. Van Lear, Jr., Am. Phys. Teacher 2, 99 (1934).
[9] G. A. Van Lear, Jr., Am. Phys. Teacher 6, 43(A) (1938).
[10] G. A. Van Lear, Jr., Am. Phys. Teacher 6, 336 (1938).
[11] Leonard T. Pockman, Am. J. Phys. 8, 64 (1940).
[12] George Lindsay, Am. J. Phys. 19, 487(A) (1951).
[13] George A. Lindsay, Am. J. Phys. 20, 86 (1952).
[14] John A. Eldridge, *College Physics* (John Wiley and Sons, Inc., New York, 1940), p. 28.
[15] Lloyd W. Taylor, *Physics the Pioneer Science* (Houghton Mifflin Company, Boston, 1941), p. 97.
[16] A. Wolf, *A History of Science, Technology, and Philosophy in the 16th and 17th Centuries* (The Macmillan Company, New York, 1935), p. 220.

A Laboratory Exercise on Determination of a Power Law by Logarithmic Plotting

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THE exercise described below was designed to illustrate the general method of determining if two variables, x and y, are related by a power law of the form $y=kx^m$, by experimentally determining pairs of values of x and y and plotting their respective logarithms. The existence of a power law connecting the variables, even over a limited range of the observed values, is indicated by a linear portion of the logarithmic plot over this range,

the slope and intercept of the line giving the values of m and k, respectively. The method is a valuable one in research and it is felt that students majoring in physics should have the experience of working on at least one exercise illustrating the technique.

The relation between the power consumed by an incandescent electric lamp and its resistance when run at various voltages has proved an interesting and informative illustration of the method over a period of years in this laboratory. Besides illustrating the principles involved, the exercise has provided an opportunity for the student to make accurate readings with ordinary electric meters, the results of which enable the instructor to check the quality of the student's work both rapidly and accurately.

The apparatus required is available in every laboratory and consists of a 115-v, 100-w incandescent lamp, a 0–150-v voltmeter and 0–1-amp ammeter, both of the six-inch, mirror-scale type, and a rheostat of about 400 ohm capable of carrying a maximum of 1 amp. The components are wired as shown in Fig. 1. In this laboratory, the experi-
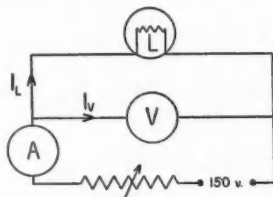

FIG. 1. Circuit for lamp measurement.

ment is carried out using a 150-v dc battery supply but a Powerstat operated from service mains may be substituted. In this case, however, care must be taken to read the meters simultaneously to avoid errors due to service voltage fluctuations. Values of the voltage across the lamp and corresponding current in the ammeter are obtained for various settings of the rheostat, the lamp being run at voltages ranging from 150 down to 20. The current I_L taken by the lamp can be computed by deducting that taken by the voltmeter I_V, the latter being calculated from the resistance of the voltmeter and its readings. The calculations of lamp resistance R and power consumed W are then carried out and the logarithms (to the base 10) of these quantities are listed and plotted. Figure 2 shows the graph obtained by a student in a recent class.

It is seen that there is a linear variation of $\log W$ with $\log R$ from a maximum power of 150 w down to about 10 w, where the linearity fails. This lower limit corresponds to the conditions under which the filament is just hot enough to emit light. The range over which there is a power-law relation is thereby determined. The straight portion of the graph may be expressed in the form

$$\log W = m \log R + \log k,$$

where the slope of the graph is the power index and the antilogarithm of the vertical intercept is the proportionality constant k. Experimental values give a slope of 3.32, the intercept (found by extrapolating mathematically) being -5.14. This wholly negative logarithm may be written as $-6.00 + 0.86$, where the mantissa is positive, the corre-

sponding value of k being 7.24×10^{-6}. Using these values of m and k the relation sought may be written:

$$W = 7.24 \times 10^{-6} R^{3.32}.$$

Students have often expressed some surprise at the discovery of a relation involving a nonintegral power-law index, realizing, perhaps for the first time, that some laws in
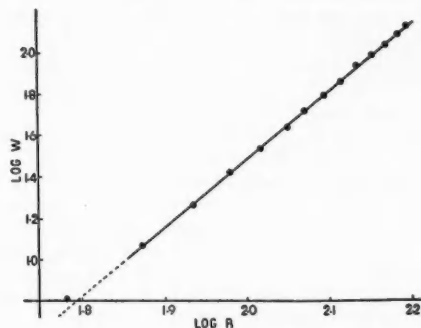
FIG. 2. Power consumption *versus* lamp resistance.

physics involve power indices which are not even simple fractions let alone whole numbers. In the writer's opinion, this surprise affords another good reason for the inclusion of an experiment of this type in the regular laboratory curriculum. In order to impress on the student that not all related quantities can be expressed in terms of power laws of this type it might be well to include an experiment in which a nonlinear logarithmic plot is obtained. A fixed length of bead chain, suspended by its ends from two points in the same horizontal plane, assumes the form of a catenary, the equation of which involves a hyperbolic function. The logarithmic plot of sag *versus* span (the latter being varied to vary the former) is not straight, thus verifying that a power law does not exist—a fact easily seen mathematically.

It is worth noting that the power consumed by a lamp at high filament temperatures is almost entirely radiated according to the fourth-power law, the resistance of metals at high temperatures being nearly proportional to the absolute temperature. Convection and conduction of heat from the filament become of increasing importance at lower temperatures. Therefore, one would expect the power law, as found above, to fail at lower temperatures. The values of resistance and power for the lamp when operated at about 2 v, for instance, give a point which lies well above the graph shown in Fig. 2.

Another exercise which may be carried out to illustrate the technique described above is the determination of the law of cooling of a calorimeter, filled with hot liquid, and enclosed in a draught-free space at constant temperature. Contrary to common opinion, the rate of cooling is found to be proportional to the excess temperature raised to some power m which has a value 1.3–1.6, approximately. The value of m is unity[1] if forced convection is employed.

[1] G. R. Noakes, *Textbook of Heat* (Macmillan Company, New York, 1947), pp. 45–46.

On Repeating Becquerel's Experiment*

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HENRI Becquerel first came upon radioactivity by wrapping a photographic plate in black paper and exposing it to the penetrating rays from a crystal of uranium salt which he placed on top of it.[1] It is a simple experiment, very direct, familiar in method by its likeness to x-ray photography, and inexpensive. As Oldenberg has pointed out, it is still valuable for classroom demonstration.[2] It can also, if undertaken casually, lead to disappointingly meager results, even with days of exposure, though Becquerel was able to obtain satisfactory images in five hours.[3] This Note outlines the conditions we have found necessary for comparable success.

1. A photographic plate has at least four times the speed of a cut film carrying the same emulsion. As Becquerel himself pointed out, a large fraction of the image is formed by secondary rays ejected from the glass,[4] and this can easily be verified if one examines the two sides of a developed but unfixed plate.

2. The plate chosen should have a high-speed emulsion with a long toe to its H. and D. characteristic. We have had good results with Eastman Super Ortho Press Plates and with their Spectroscopic Plates Type 103–O (both available commercially). The O sensitizing is especially convenient as it can be worked under the relative brightness of a Wratten No. 1 safelight.

3. Development should be prolonged for high gamma, but the choice of developer is not critical.

4. The uranium preparation should have a flat surface and well-formed edges so as to produce a sharply defined image on the plate. We have used a square box, 1 cm on edge, folded up from onion-skin paper and filled with black uranium oxide.

5. The plate when developed and fixed should not be treated as a transparency but laid on a sheet of white paper and viewed or projected as an opaque object.

Under these conditions, with a half-hour's exposure, using a warm developer and a rapid fixing bath, we have obtained an image of projection quality well within the time span of a single lecture. This means that if adequate darkness can be obtained (either with window shutters or a dark box on the lecture table), the experiment can be carried out in an unbroken session, entirely under the eyes of the class.

* Presented before the American Association of Physics Teachers, New York, January 28, 1954.
[1] H. Becquerel, Compt. rend. 122, 420 (1896). See also W. F. Magie, *A Source Book in Physics* (McGraw-Hill Book Company, Inc., New York, 1935), pp. 610–613, for excerpts from this and other papers.
[2] O. Oldenberg, Am. J. Phys. 20, 111 (1952).
[3] H. Becquerel, Compt. rend. 122, 501 (1896).
[4] H. Becquerel, Compt. rend. 132, 734 (1901).

LETTERS TO THE EDITOR

Space Charge between Parallel Plates

IN a recent paper Copeland and Sachs[1] assert that "for parallel planes and complete space charge Langmuir[2] first established the $\frac{3}{2}$-power law for the dependence of current on potential difference." As a matter of fact, Child[3] considered the parallel plate case and derived the $\frac{3}{2}$-power law more than two years before Langmuir.

No one questions the independence of Langmuir's work or the clear application of his analysis to this and more general thermionic diodes, but the prior derivation of the $\frac{3}{2}$-power relationship under space-charge conditions by Child should not be ignored.

[1] Paul L. Copeland and Lester M. Sachs, Am. J. Phys. 22, 102 (1954).
[2] Irving Langmuir, Phys. Rev. 2, 450 (1913).
[3] Clement D. Child, Phys. Rev. 32, 498 (1911).

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Paradox of the Stillson Wrench

I NOTE that in the April issue of the Journal, Mr. Julius S. Miller calls our attention to what he terms "The Paradox of the Stillson Wrench." Mr. Miller apparently believes that a dilemma exists in the behavior of the wrench.

Consider the machining of the teeth on the movable jaw; it is done in a way that gives the teeth a negative slope, relative to the normal motion (clockwise) of the wrench when in use. The purpose of the negative slope is quite obvious—the jaw is designed to bite into any surface irregularity it meets. These surface irregularities, I submit, are responsible for the "birth" of the frictional force that allows the movable jaw momentarily to stop sliding around the pipe. At this instant, the pivot enters the problem, with the "fixed" jaw pivoting and biting in.

To substantiate my contention that it is surface irregularities that give rise to the normal force needed to allow the wrench to function, I tried applying a small Stillson wrench to various smooth objects. In those cases where genuine smoothness was encountered, i.e., on glass tubing, the wrench cannot be made to function unless an outside agent creates and keeps in being the needed normal force. On those objects which are smooth but soft, as for example, polished steel tubing or rod, the wrench initially slips, but again can be made to function correctly if an outside agent supplies a normal force initially. In this case, once the teeth have been forced into the material slightly, the wrench functions as usual.

In the case discussed by Mr. Miller in which a wrench once tightened is released, and then retightened (by the application of torques only, not by adjustment of the jaw

spacing) the movable jaw again will slip if the wrench is moved counter-clockwise far enough. However, if the wrench is merely loosened and the tightened immediately, the teeth of the movable jaw are still engaged in the pattern they originally impressed into the surface. As the clockwise torque is increased, the only part of the wrench that can move, relative to the surface, is the "fixed" jaw. Hence, the wrench will once again function as usual.

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Observations on a Pile Driver

WHILE in New Orleans in recent years I had frequent occasion to witness the driving of piling, both wooden and hollow steel. The terrain there is so very boggy that no sizable structure can be erected without providing a new solid footing. Now in the driving of a wooden pile a heavy metal cap (not unlike a huge thimble) is sometimes placed over the upper end on which the hammer drops. The obvious purpose of this cap, if one seeks quickly for a reason, is to protect the pile from splitting at the blow of the hammer. It occurred to me to ask the engineers in charge if the heavy steel cap contributed anything useful to the driving of the pile; that is, did the pile sink farther under a blow with the cap on than it did with no cap atop the pile. The answer generally was that the cap served only to protect the head of the pile. It was further speculated that the cap decreased the depth of penetration per blow since it added dead weight to the pile which had then to be moved. On several isolated occasions where piles were driven without a cap I detected that the collision was sensibly inelastic, whereas with the cap the collision was measurably more elastic.

Now in the treatment of collision processes in analytical mechanics I try to bring out the *practical* consequences of the equations we deduce and some interesting conclusions are forced upon us. At least two very distinct situations arise, best illustrated by these examples: the shaping or forging hammer on the anvil, on the one hand, and the pile driver on the other.

In the forging or shaping we wish to deform, hence e,

the coefficient of restitution, is practically zero and most of the energy is dissipated. The opposite is true in the driving of a pile, where we wish to deliver kinetic energy to the pile. Analytically, the results from the usual standard equations[1] show the *loss* of energy to be

$$\frac{(1-e^2)}{2} \frac{M_1 M_2}{M_1 + M_2} U_1^2,$$

where M_1 is the mass of the moving body, M_2 the mass of the stationary body, and U_1 the velocity of the hammer at impact. Now what proportion of the original energy is lost? Clearly the amount

$$\frac{1-e^2}{1 + M_1/M_2}.$$

In the forging, since e is practically zero, this becomes $M_2/(M_1 + M_2)$ parts of the original energy. Obviously, then, the forging or shaping hammer M_1 may be light (should be), and the anvil M_2, heavy. This is consistent with experience and intuition. In the pile driving, however, the proportional loss can be decreased by having M_1, the hammer, very great compared with M_2, the pile. This, too, is consistent with what we observe in pile driving. But our advantage is also enhanced by having e as large as possible. This we knew all along since it characterizes highly elastic collisions. Accordingly, pile drivers should make their pile driving elastic operations! Indeed, it is an elegant exercise to show that if the impact could be made purely elastic ($e = 1$) the pile would receive *four* times as much energy[2] as it does when $e = 0$.

It occurs to me finally to answer a paradox which I often raise in class discussion. It concerns the driving of a small stake or post as one would on occasion do in his own backyard. Is it not easier to drive the stake with a heavy hammer (swung even gently) than with a light one, even though the light one is swung with enormous velocity, thus giving it enormous energy? It is like saying: it's not the temperature; it's the humidity!

[1] Timoshenko and Young, *Engineering Mechanics* (McGraw-Hill Book Company, Inc., New York, 1940), p. 334 ff.
[2] In this discussion the impulse of the ground reaction on the pile *during* the impact is neglected. This invokes no serious error.

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ANNOUNCEMENTS AND NEWS

Book Reviews

Dimensional Methods and Their Applications. C. M. FOCKEN. Pp. 224+VIII. Edward Arnold and Company. London, 1953; St. Martins Press, Inc., New York, 1954. Price $6.00.

The author states that he set out to write a book that would not seem heavy or unwieldy to engineers and practical men. This purpose would have been more easily attained if the book had not been intended to be also a

critical appraisal of the present position of the controversial subject of dimensions. The discussion of the conflicting points of view of various authors is likely to confuse a beginner.

The formulation of principles in the first half of the volume follows in general Bridgman and especially Dingle, but is not always very clear. Consider, e.g., the following fundamental definitions on p. 21: "the result of the measurement, consisting of the numeric times the unit, is called the *magnitude* of the corresponding physical quan-

tity. It has previously been stated that a physical quantity is specified precisely by its magnitude, which consists of a numeric and a unit. The symbols which appear in physical equations represent the magnitudes of the physical quantities concerned. For example, the equation $v = u + at$ is a relation between certain numbers. These numbers are the numerics, which are the result of measurement." As in the above equation one is often puzzled whether the symbols in equations represent magnitudes or numerics. The product of a numeric and a unit, and the products of units, are probably understood to be symbolic products for which the usual rules of multiplication are valid, but such an explanation is not given.

The author follows Bridgman in some points, which are not quite satisfactory. An equation is called complete if it is true in all units, but sometimes also if it is unit-free, i.e., invariant, while the second property follows from the first only if the equation is the only relation between the quantities involved. The theorem "If A, B, C, $\cdots$ are fundamental magnitudes defining a derived quantity S, then relative magnitude of the quantities S has absolute significance only if $S = kA^aB^bC^c\cdots$, where k is a constant" is not correct if some of the magnitudes, e.g., A and B, are of the same kind, since it is then possible that $S = kf(B/A) \times A^aC^c\cdots$, where $f(x)$ is an arbitrary function. Two proofs of this theorem, according to Bridgman and O'Rahilly, are given, but a proof of the pi theorem is not presented, although it would not be much more difficult.

The dimensions of thermal, electrical, and magnetic magnitudes are discussed very neatly. Electric charge is recommended as the fourth fundamental magnitude, and the unit of another quantity, e.g., resistance or permeability, as the most appropriate fourth fundamental unit.

The applications of dimensional methods cover the second half of the book. One chapter deals with physical applications, which include theoretical considerations on physical constants, chance dimensional identities, and applications of dimensional analysis to ultrahigh frequency electronic devices, atomic physics, and nuclear physics. Another chapter deals with engineering applications and discusses model experiments in fluid mechanics, heat transfer, geology, and chemical engineering. About a hundred problems with answers accompany these two chapters.

The book gives a satisfactory survey of the literature on dimensions which has been published in English, and contains much valuable information for everybody interested in the subject.

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Methods of Theoretical Physics, Parts I and II. PHILIP M. MORSE AND HERMAN FESHBACH. Part I, 1037 pp. Part II, 997 pp. 16×24 cm. McGraw-Hill Book Company, Inc., New York, 1953. Price: $15.00 each part, $30.00 a set.

This massive work is based on a course given by the authors to physicists and engineers at about the first year graduate level at the Massachusetts Institute of Technology. A first course in differential equations or advanced calculus is presumed, but it is intended that the book should be complete for the material in vector and tensor analysis, linear differential and integral equations, complex variable theory, and the theory of Fourier and Laplace transforms, to cover those parts of mathematical physics which are concerned with the classical theory of fields. An introductory treatment of quantum-mechanical theory, the Schrödinger wave equation, and the Dirac equation is included.

Naturally there is much overlapping with other sources, such as Sommerfeld's *Partial Differential Equations in Physics*, the well-known text by Courant and Hilbert, *Methods of Mathematical Physics*, which is now being issued in an English translation, and the somewhat oldish but still valuable book by A. G. Webster, *Partial Differential Equations of Mathematical Physics*. However, Morse and Feshbach's book differs from all of these both in its choice of material and in its manner of treatment, being strongly infused with the characteristic style and breadth of interest in applied problems for which the authors are known by their contributions in the periodical literature.

In attempting to assess the position of this book with respect to other available literature it is important that it be considered in the light of the authors' own philosophy in writing it. The following quotations from the text give the major points. From the Preface: "This is not to say that the work is a text on mathematics, however. The physicist, using mathematics as a tool, can also use his physical knowledge to supplement equations in a way in which pure mathematicians dare not (and should not) proceed. He can freely use the construct of the point charge, for example; the mathematician must struggle to clarify the analytic vagaries of the Dirac delta function. The physicist often starts with the solution of the partial differential equation already described and measured; the mathematician often must develop a very specialized network of theorems and lemmas to show exactly when a given equation has a unique solution. The derivations given in the present work will, we hope, be understandable and satisfactory to physicists and engineers, for whom the work is written; they will not often seem rigorous to the mathematician."

From page 265: "We have seen the same fields and the same differential equations turn up in connection with many and various physical phenomena. We find, for instance, that a scalar-field solution of Laplace's equation can represent either the electric field around a collection of charges or the density of a diffusing fluid under steady-state conditions or the velocity potential of a steadily flowing incompressible fluid or the gravitational potential around a collection of matter, and so on. From the point of view of this book, this lack of mathematical originality on the part of nature represents a great saving in effort and space. When we come to discuss the solutions of one equation, we shall be simultaneously solving several dozen problems in different parts of physics."

From page 1173: "Moreover, we shall be primarily concerned with the application of the more advanced techniques of computation. The simpler tools of theoretical

physics have been touched on, here and there, in the earlier parts of this work, and they are elucidated in many well-known treatises. It is felt that the less familiar methods need the exposition here. As a result, of course, the remaining chapters of this work will be rather heavy going for the casual reader, and in many places, it will be hard to see the forest for the trees. It is hoped that, from the very mass of details, the less casual reader will eventually sense the pattern of technique and will begin to acquire that semi-intuitive "feel" for a new problem which is so difficult to teach but which is so useful a faculty for the theoretical physicist."

The text itself bears out the deduction which one would draw from these quotations that the authors are concerned primarily with the techniques of extraction of explicit formal solutions for the more complex types of boundary value problems encountered in mathematical physics. The style of writing reflects the origin of the book in lecture notes and enforces the result that it is primarily a working text for systematic study, rather than a reference work. A user who has not put in his apprenticeship by a study of the text will be likely to suffer at times from a severe case of frustration, as the authors recognize. The publisher's offer to sell the volumes separately cannot be taken as a realistic implication that they can be read independently. Fortunately, each part has been supplied with a complete table of contents and a complete index. It would have been well if different type fonts had been used in the index to distinguish the pages of the two volumes since pagination is consecutive. If one is using the index constantly it is convenient to remember that Part II starts on page 999, so that the number 1000 is a convenient division point.

It would serve little purpose for the reader if the attempt were made here to review the different mathematical methods treated in the book. These are of standard character, for the most part, and the major purpose of the book is pedagogical rather than inventive. The two features which the reviewer has found of the most interest are the detailed discussion of separability of differential equations by appropriate choice of the coordinate system (Chapter 5), and the effort which is made to improve the standard of treatment of perturbation calculations (Chapter 9). This makes accessible material of considerable practical importance and interest which has not been available heretofore in book form. Although it would be too much to expect that the authors could establish a properly convergent perturbation theory for the variety of problems encountered in quantum-mechanical theory, their discussion of methods of improving convergence (assuming it to exist at all!) is a welcome relief from the trite attitude adopted towards this problem in much current literature.

An innovation of possibly great pedagogic value is the use of paired drawings for stereoscopic viewing. It is difficult for one who has been long accustomed to the visualization of plane drawings as 3-dimensional figures to appreciate the value which these may have for a beginner, but the reviewer is prepared to believe that this may be one of the most significant new features of the book.

Since the authors are so careful to indicate the limitations of their treatment from the point of view of the pure mathematician, it might seem ungracious to tax them with lack of rigor in some parts of their analysis. However, not being a pure mathematician, the reviewer feels the more free to question one or two of the methods used which he finds it difficult to accept. The reason for doing so will be apparent when one reflects that owing to the great demands which the study of this book will place on a student, to many it must become the court of last resort as a mathematical reference. The authors therefore cannot escape the requirement of a reasonable mathematical rigor merely by defining it away, but must submit to a legitimate examination of the methods used in their book as a basis for further analysis.

Probably that feature of the mathematical treatment which the reviewer finds the least appealing is the use of the Dirac delta function as a standard technique in the discussion of Green's functions. The authors open themselves at this point to a serious charge of the deliberate introduction of an ultimately unmanageable technique for the sake of a temporary and specious generality of method. We all know, of course, that the delta function has become accepted dogma among the younger generation of quantum mechanists, that it is widely used in the current periodical literature of mathematical physics, and that some mathematicians employ it as a convenient shorthand. No doubt there is ample excuse for the use of symbolic methods in discussions in which the author is quite competent to prove his results otherwise, or in which the subject matter has a certain exploratory and ephemeral character. But it is a quite different matter to present such methods to students for use on problems which are new to them and where their inexperience gives them little recourse to alternative procedures when the calculation bogs down on some subtle point of interpretation. This is particularly the case when systematic alternative procedures are known, are readily found in the literature, and are not infrequently simpler than the indefinite arguments which are almost certain to attend the use of the delta function.

It can be admitted readily that there is a range of theory in which it is relatively easy to justify the formal use of the delta function and to lay down unambiguous rules for its use which make it a profitable, if not indispensable, tool. But when one attempts to make use of it in the formulation of general Green's functions some pathological difficulties arise which one cannot expect students to unravel. These troubles appear to arise from two major causes: (a) owing to the different connections in which the delta function arises one needs many different "representations" for it which are difficult to systematize and codify, and (b) the use of the delta function leads one to overlook the fundamental distinction which is to be drawn between a 4-dimensional Fourier integral and an eigenfunction expansion in terms of plane waves. Apart from the present text the only other one known to the reviewer which makes such a consistent attempt to elevate the formal use of the delta function to the status of a universal technique is that of Ivanenko and Sokolov [*Classical Theory of Fields* (Russian), Moscow, 1951]. Ample evidence that even the

devotees of the delta function find it difficult to keep under control can be found in both of these books as well as in the current periodical literature.

Difficulties of type (a) are often "removable" in individual cases by a careful arrangement of the argument, and their danger lies in the fact that they are apt to occur unexpectedly in a calculation and to lead to confusion in a way which is difficult to trace to its origin. As a particularly obvious example the discussion on page 839 is noted. Here a half of a printed page is used on the "proof" of the relation $\delta(R)/R = -\delta'(R)$. If one accepts the delta function as being differentiable the desired result follows from a differentiation of Dirac's identity $R \cdot \delta(R) = 0$. The procedure used by the authors would be of greater appeal if they were to make a more consistent use of limiting processes in the use of the delta function.

An example of a more serious difficulty of type (b) occurs on page 1433. Here the problem is to obtain the solution of d'Alembert's equation for which (1) the source function vanishes identically for $t < 0$ and is prescribed arbitrarily for $t > 0$, and (2) the solution is "subject to no boundary conditions except that the wave be outgoing in all directions at infinity." A formal solution is obtained by a method which the reviewer finds quite unintelligible owing to the imprecision in the definition of the Green's function discussed on pages 838–40. Looking at the result given by the authors it is evident that the most direct path to it would be by a representation of the desired solution in terms of a 4-dimensional Fourier integral. The general problem has been so treated and its subtleties discussed in a more straightforward manner than is used here by G. A. Schott (*Electromagnetic Radiation*, Cambridge University Press, 1912, Chap. II).

The reviewer also finds the introduction of the notion of Lebesgue integration and its use in the "proofs" of the important theorems concerning Fourier transforms, all without benefit even of an explicit definition of the Lebesgue concept of measure, to be quite unsatisfactory. The "practical effect" of the new definition of integration is indicated to be that it permits the integration of functions which oscillate with infinite rapidity or which have a noncountable set of discontinuities. Has no engineer or physicist in the authors' classes ever questioned *why* the extension to such recondite functions is considered to be necessary for applications to physical problems? There does not, in fact, seem to be any necessity for dispensing with Riemann integration for the level of solubility of problems achieved in this book so that the course followed in their discussion by the authors only introduces an unused generality at the potential cost of serious misunderstanding by the thoughtful student.

The reader is likely to wish at times that the authors had sacrificed some of the heavy analytical work for the sake of a more considered treatment of the meaning and limitations of the physical theories concerned. A case in point in which the authors have allowed their desire for uniformity and generality to cloud their judgment occurs on page 139 where the Klein-Gordon equation is introduced as governing the motion of a string which is subjected to a special kind of stiffness arising from the medium

in which it is embedded. After the analysis of this mechanical problem one is suddenly confronted with the following statement: "This equation is called the Klein-Gordon equation when it occurs in quantum mechanics. We note that, if c is the velocity of light, this equation is also invariant in form under a Lorentz transformation, as is the wave equation, so that the solutions of the equation behave properly with regard to the space-time rotations of special relativity." This juxtaposition of ideas from ordinary mechanics and from the special theory of relativity is startling, to say the least. A moment of reflection shows one, of course, that it is only the *differential operator* which is invariant under Lorentz transformations, and that there is no justification for the statement that the equation is invariant unless one knows that the dependent function has the proper relativistic transformation property. This can be assumed to be the case for the quantum-mechanical theory and the conclusion becomes true, but for the embedded string it is false.

Another instance which requires notice occurs in the discussion of the relationship between the coordinate and momentum space representations in quantum mechanics. It is stated at a number of places in the text that one can obtain the differential equations for the wave function in momentum space by making the formal operator replacement $q \rightarrow i\hbar \partial/\partial p$, $p \rightarrow p$, in the Schrödinger Hamiltonian function. It is well known, however, that this procedure is valid generally only for special forms of the Hamiltonian. If it is tried for the case of the hydrogen atom, for instance, some real difficulties become apparent at once. It is interesting that for this problem the authors do not attempt to use the rule but obtain the momentum space wave functions by direct Fourier transformation on page 1679, as was done originally by Podolsky and Pauling. No indication is given of the derivation of the appropriate differential equation in momentum space by Hylleraas, or of the group-theoretical treatments by Fock and Bargmann which have given a beautiful clarification of the mathematical nature of the problem.

The examples which have been indicated here as making difficulty for the reader are neither isolated cases nor are they typical of the text. The reviewer would emphasize, however, that since this book is likely to have a significant influence on the training of applied theoreticians in the next few years, and so will determine the attitude of many people towards applied mathematics, it is important that the obscurities be eliminated wherever possible in future editions. The reviewer is not convinced personally that the amount of mathematical theory covered in this book should be taught in physics courses, but it must be admitted that while we have a superabundance of elementary textbooks and more than a few excellent advanced mathematical monographs on the problems of mathematical physics, the level of instruction and study for which Morse and Feshbach have written their book has been sadly neglected.

The quality of the printing and editorial supervision of the mathematical work in the book is very high. The reviewer has not attempted to check formulas deliberately to gather statistics on errors, but judges that these will be

relatively few considering the fact that this is a first edition and that the analytical work is very heavy. A number of minor typographical slips have been detected in reading the text, but few of these are likely to cause the reader serious difficulty. One linguistic comment seems to be in order. The persistent substitution of the auxiliary verb *may* for *can* has become almost a pathological characteristic of modern scientific writing, leading usually to a loss of clarity in meaning. The authors indulge in this practice quite freely and a spot check of the text indicates that if one systematically replaces *may* by *can* there will be a distinct improvement in the sharpness of meaning in the great majority of cases. Publishers' editorial staffs please take note!

E. L. HILL
University of Minnesota

Experimental College Physics. MARSH W. WHITE AND KENNETH V. MANNING. Third edition. Pp. 347+XII. McGraw-Hill Book Company, Inc., New York, 1954. Price $5.00.

Since this is a laboratory manual of experiments for use in a general college physics course the reviewer will apply himself to the following questions. 1. To what extent do the experiments meet the needs of such courses? 2. What auxiliary material is offered for the use of student and instructor? 3. What distinctive approach do the authors have to the problems of laboratory teaching?

1. The book contains 76 experiments collected into 45 chapters. Most of the experiments are familiar in that they are the usual ones performed in a general physics course and the equipment used will be recognized since practically all the pictures show the names Cenco or Leeds and Northrup. Many chapters have an elementary experiment in which one might consider that the student is observing the phenomenon under consideration rather than making measurements of it. They are followed by experiments in which the emphasis is on measurement. Examples are 12.1, in which a rubber ball attached to a string is made to revolve rapidly enough to support a weight hanging from the same string; 12.2, the familiar Cenco centripetal force experiment; 13.1, elongations of a spring and a rubber tube; and 13.2, Young's modulus for steel. There are other simple experiments which can add much to the understanding students have of the phenomena involved, e.g., 15.1, an interesting method of determining pressures at various depths by finding how far a long cylinder with a flat bottom will sink various weights put inside, or 40.1, refraction and reflection by means of the optical disk. The idea of using such qualitative experiments, especially for some students, is good. It might be expanded to areas where the subject is especially in need of illustration, such as in observing the phenomena of diffraction by single, double, and multiple slits.

There are many good experiments of the average degree of difficulty used in a general physics course. It is always possible to suggest additions. For example, there is no experiment on heat conductivity. The reviewer would like to see a few more difficult experiments. For example, Chapter 14 has two fairly easy experiments in simple

harmonic motion (the first one having the interesting complication of determining experimentally the fractional part of the mass of the spring which must be introduced in the period equation to get satisfactory results) but there are no experiments in which the experimental procedure is more complex, as in Kater's pendulum, the method of determining the period by coincidence pendulums or by successive approximations.

2. There is a brief treatment of the underlying theory for each experiment and there are so many stimulating problems and questions that the instructor has a choice of the ones he wants to emphasize. It is good to find questions in a laboratory manual which have no "right" answers; students have to learn to use judgment and common sense in the laboratory. One of the intangible values which the student is expected to get from the laboratory work is "Respect for the Prestige and Reliability of Authority" (page 2). "Ability to Evaluate the Reliability of Authority" might be a more suitable term, because too much emphasis on prestige may lead to difficulties. Question 4, page 10: "A group of people witness a traffic accident. A housewife and an engineering student are required to testify concerning the time sequence of of events, the relative speed of the vehicles, and similar observational details. In which testimony might the court place the greater confidence? Why?" There is plenty of opportunity here to consider the many factors to be considered in evaluating an authority (or a witness) other than that he or she has a certain classification such as "housewife" or "engineering student."

There is a good introduction dealing with errors, significant figures and graphical analysis. The appendix contains all the necessary mathematical and physical tables, a section on the *Adjustment of the Mercury Barometer*, one on *Method of Focusing a Reading Telescope* and one on *Conventional Symbols Used in Diagrams in Electric Circuits.*

3. In trying to interpret the approach to laboratory teaching which is represented in the book we take first the statement in the Preface. This book "is not merely a set of directions for performing experiments · · · a studious attempt has been made to design the steps in the manipulation of the apparatus so that the reasoning powers of the student are utilized and developed to the maximum extent." The book gives a set of detailed directions for performing the experiments and sufficient discussion of the underlying theory so that the student can follow the logical development. The most obvious places where the student is expected to use his powers of reasoning are in following these developments. Only in the evaluation of the errors in the experiment does he get a chance to use his own judgment and there he usually follows a routine procedure indicated in the preface. This is typical of physics laboratory manuals, good ones, but one wonders if more development of the reasoning powers and more understanding of the phenomena would result if he were not told in such detail just how to proceed and what the theory of the experiment is and were given instead a problem and asked to find his own solution. He might afterwards be asked to see if his solution is consistent with those that come from theory. For example, in Chapter 12 centripetal

force relationship is developed and in experiment 12.2 the student is asked to calculate *f* from the conditions of his experiment and compare it with the force needed to stretch the spring. He might do more reasoning if he were asked to measure the force for various values of the angular velocity (in revolutions per minute) and then find an empirical relation between them. He could then be asked to find if there is a theoretical conclusion with which he can compare his results. This does not differ greatly from the usual method but it leaves more opportunity for the student to use his reasoning powers and to find things for himself.

Since this is a manual of experiments designed for instructional purposes the reviewer is glad to see that the *object* of the experiment is frequently stated in terms of the educational result to be attained. This might well be carried much further. For example, the *object* of experiment 5.3 is given "To study the motion of a freely falling body and in particular to measure *g*, the acceleration due to gravity." There are better ways of determining *g*, even in the elementary laboratory, so the *object*, the reason for doing this experiment, is something else, perhaps to practice certain techniques in the interpretation of data (taking first and second differences, finding an average by means of a straight line curve, etc.) and *incidentally* to make a determination of *g*. Similarly 13.2 on Young's modulus might well have as a major part of its *object* practice in the use of a laboratory telescope and of the optical lever. The value of Young's modulus obtained is incidental to the instructional aspects of the experiment.

These comments should not mask the fact that the reviewer finds this a well-prepared laboratory manual with a large number of experiments suitable for a general physics course, with all the auxiliary material that is normally needed in doing these experiments and with an approach which encourages the student to consider the educational outcome of the experiment as well as the numerical results obtained.

G. E. Owen
Antioch College

Introduction to Electric Theory. R. G. Fowler. Pp. 390 +xii, Figs. 241, 15×23 cm. Addison-Wesley Publishing Company, Inc., Cambridge, Massachusetts. Price $7.50.

The advance of physics depends not only on new discoveries but also on new textbooks whereby the rising generation of physicists may excell their teachers. The task of reinterpreting classical electrical theory for junior students has been undertaken by the present author with acknowledgments to Livens, Jeans, Stratton, Maxwell, and the Coulomb committee. The result is a text for students who have studied general physics and ordinary analytic geometry and calculus, suitable for a year course.

As befits a book on theory, experimental findings which form the basis for theories of electricity and magnetism are quoted, and there is continual emphasis on the physical content of the mathematical treatments employed, but experimental details and descriptions of apparatus are

generally omitted. Historical perspective is provided by giving the dates of important discoveries, and the names of the physicists who made the discoveries.

Vectors are used "in the same spirit that they are presented to students of general physics, and without the graceful but impractical trappings of a complete vector notation." Even though standard vector notation is not used in the development of the subject, important equations are summarized in this notation, following their derivation in component form.

The first four chapters deal with electrostatic fields in vacuum and in matter. The concept of flux of a vector field is introduced by speaking of water in a stream flowing across a "control surface" dividing upstream from downstream. Sparing use is made of lines of force, to avoid "saddling the student with too vivid a sense of their reality." The author tries not to fall into the trap of using undefined terms; thus even the technique of disposing of charges by "grounding" is definitively explained.

In Chapter 5, current is defined as the rate at which positive charge is passing across a given control surface, used in the sense mentioned above. It is emphasized that a current does not flow, although even scientists use the expression "current flow." The Franklin sign convention is explicitly retained and the argument for reversing it is rejected on the ground that there would still be situations in which charges move "backward," in conduction through electrolytes and through gases, and in the case of displacement currents. The term "electromotance" is used for the work done per unit charge, noting that "electromotive force" is unsatisfactory for this potential-like quantity, "so that many authors in despair have adopted the practice of abbreviating it to *emf* in the hope that the misnomer will pass unnoticed." This remark might have more force if "force" were used only in the sense of Newton's laws; perhaps we should force ourselves to avoid contrary usages.

Magnetic fields and magnetic materials are treated in Chapter 6, with proper skepticism about the existence of free magnetic charge. "An isolated imaginary magnetic charge · · · can be approximated in practice by making a very long magnet like a knitting needle · · · —but it is much easier to obtain in fancy." The magnetic field of a current is taken up in Chapter 7. The word magnetomotance is used to replace *mmf*, and the limitation of the concept to large, thin toroids is stressed. Electromagnetic forces and electromagnetic induction are covered in Chapter 8 and stress and energy in the electromagnetic field in Chapter 9.

The electrical properties of matter are so briefly discussed in Chapter 10 as to risk oversimplification. The quantum nature of matter is touched on with simply a caution: "In most respects, and especially in the study of radiation, it is completely false to apply the arguments developed from macroscopic electrical theory to atomic and molecular systems."

Constant current circuits are covered in Chapter 11, although Kirchhoff's first law first appears in Chapter 5. Varying currents in linear circuits are treated in Chapter 12, and periodic currents in Chapter 13. The author hopes

that "instructors will assign problems in these two chapters sparingly and with charity, realizing that they will, in many cases, require considerable time."

The final chapter takes up electromagnetic radiation and Maxwell's equations. There is an appendix on units in electricity, which is a clear and temperate exposition of reasons for selecting the mks system. A list of symbols, answers to odd-numbered problems, and an index complete the book.

The style is persuasive and is enlivened with instructive figures of speech. The workmanship in writing, editing, and publishing is admirable. Only three or four misprints were noticed on first reading. The test of a text is its usefulness to students, and to this reviewer it appears likely to pass that test with distinction.

HAROLD P. KNAUSS
University of Connecticut

Nuclear Forces. W. HEISENBERG. Pp. 225. Figs. 40. 12½ by 19½ cm. Philosophical Library, New York, 1953. Price $4.75.

This book is based on a series of lectures given by the author under the auspices of the Association of German Electrical Engineers. It is designed for readers who, while they may have had no training in theoretical physics, nevertheless have some degree of familiarity with modern physical ideas. On the other hand, a knowledge of anything but the most elementary mathematics is unnecessary, the book being almost completely descriptive in nature.

The book begins with a short history of the views about atoms held in antiquity and a summary of modern atomic theory, up to the end of the nineteenth century. The second chapter is devoted to a survey of the development of atomic theory in the twentieth century, tracing this development from Rutherford's model of the atom through Bohr's theory to quantum mechanics. This survey provides an introduction to the main subject of the book, the structure of the nucleus, which occupies the remaining six chapters.

The author begins his treatment of the nucleus by using the information obtained from radioactivity and artificial nuclear transmutations to deduce the constituents of nuclear matter. He continues with a discussion of the normal states of atomic nuclei, introducing the concept of the binding energy of nuclei and examining, on the basis of a liquid-drop model for the nucleus, the types of nuclear energy which contribute to the binding. This is followed by a general description of nuclear forces, including their short range nature, their exchange character, and their property of saturation. These properties are then used to account for the systematics of nuclear stability. The author then deals with the subject of nuclear reactions, discussing, by means of classical analogies, the processes of alpha and beta emission and spontaneous fission. Following this is a chapter devoted to the tools of nuclear physics describing the instruments used in the detection of particles and the machines used to accelerate them. The book concludes with a discussion of the practical applications of nuclear physics.

The author has done an excellent job of conveying many

physical ideas without the use of mathematics. His treatment of alpha radioactivity is especially good in this respect. However, the book has one serious shortcoming. The material presented was last revised in 1948 and consequently the book does not contain the many fundamental developments in nuclear physics which have taken place since then. Thus, for example, the role of the π meson in nuclear forces is presented only as a tentative suggestion. There is no discussion of the shell model which has played such an important role in helping us understand the structure of nuclei. The chapter on the tools of nuclear physics contains no description of many of the tools which have such widespread use today, such as the betatron and synchrotron in the field of accelerators and the scintillation counter in the field of detectors. And, of course, there is no account of all the new mesons which have been discovered in recent years. As a result of the fact that such basic material is missing, the book provides only an incomplete introduction to the field of nuclear physics.

CARL GREIFINGER
University of Pennsylvania

Space Travel. KENNETH W. GATLAND AND ANTHONY M. KUNESCH. Pp. 205, Figs. 25, 14×22 cm. Philosophical Library, Inc., New York, 1953. Price $4.75.

The field of rocket science and space travel is rapidly becoming a highly competitive one in which the original science fiction writers are now being crowded out by such names as Willy Ley, Wernher von Braun, and Donald Menzel, to mention but three of many. Authoritative books on rockets and on the possibilities of space travel now sell for as little as 75¢ on the magazine stands. The present book, intended for popular consumption, makes no outstanding contribution to this field though the reader will find the figures and tables as reliable as they can be in a subject on which there is little agreement even among the experts.

The first half of the book is taken up with the history of rocket development in Germany, Great Britain, and the United States and gives the impression of being little more than a collection of notes, reports, and stories obtained from the files of the several rocket societies interested in rocket propulsion and interplanetary travel. The second half of the book discusses the possibility of such projects as an artificial satellite, a trip to the moon, atomic rockets, and even interstellar trips which might consume several hundred years in passage. This half is also well supplied with tables of technical data on the rocket vehicles which would be necessary for such space travel. It should be understood that the figures given are based on estimated motor performances which may well be possible but which have not yet been achieved. No definite estimate of the time needed to organize a trip to the moon is given by the authors though others have estimated the expenditure of time to be anywhere from 25 to 75 years and the cost to be at least four billion dollars if we start now.

The physics teacher who reads the book will be annoyed by the many small discrepancies, the inexact statements, and the tendency toward slang expressions such as "ion-

drive," "milligee," "g" loss, etc. For example we find the statement that the earth exerts a *pull* of 6.95 miles per second and again that the rocket's *velocity* may exceed the opposing *gravitational factor*. Later on we find the suggestion that in order to prevent loss of heat by radiation from a vehicle traveling through space it will be necessary to provide double walls with a *vacuum* between. It is doubtful whether such precautions will be effective when the vehicle is already traveling in a near-perfect vacuum. Also, in a description of a space ship in flight between the earth and moon the latter's gravitational pull is given as one-sixth that of the earth's. This factor, of course, depends on relative distances and is approximately 1/80 at a point equally distant from each.

On page 149 the ultimate velocity of a rocket is referred to as its "characteristic velocity." This term is usually reserved for a more complex function expressing the figure of merit of the motor alone. The equation given for final velocity is essentially correct, however, and gives an interesting insight into the relationship between exhaust velocity and mass ratio. No book on space travel is complete without pictures and this one is well supplied with clear and interesting photographs of almost all of the nonclassified rockets produced to date.

<div align="right">

RALPH B. BOWERSOX
Jet Propulsion Laboratory

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Publications Received

The descriptive phrases are quotations from publishers' advertisements or from authors' prefaces.

Acoustics. T. M. YARWOOD. 346 pp. St. Martin's Press, Inc., New York, 1954. $3.50. For those preparing for all stages of the General Certificate of Education examination.

Annual Review of Nuclear Science. Volume 3. 412 pp. Annual Reviews, Inc., California, 1953. $7.00. To encourage specialized research men to be currently aware of progress in related fields.

Applied Electronics. TRUMAN S. GRAY. 881 pp. The Technology Press (Massachusetts) and John Wiley and Sons, Inc., New York, 1954. $9.00. The major aims in the revision have been: to improve and clarify details; to bring the coverage up to data; and to include new developments such as the transistor and its applications.

Climatic Change. HARLOW SHAPLEY. 318 pp. Harvard University Press, Cambridge, Massachusetts, 1953. $6.00. Why climate changes and what its changes do to earth and living things.

Complex Variable Theory and Transform Calculus. (Second Edition). N. W. McLACHLAN. 388 pp. Cambridge University Press, New York, 1953. $10.00. A modern treatment of the so-called operational method, and its application to problems in various branches of technology.

Crystal Growth and Dislocations. A. R. VERMA. 182 pp. Academic Press, Inc., New York, 1953. $5.00. Recent developments in the field of crystal growth, with special reference to the spiral growth of crystals.

Design of Relays. (Monograph 2180). 314 pp. American Telephone and Telegraph Company, 1954. Devoted entirely to the analysis and measurement of relay performance, and to the economic considerations which govern optimum relay design.

Dimensional Methods and Their Applications. C. M. FOCKEN. 224 pp. Edward Arnold and Company (St. Martin's Press Incorporated, New York), 1954. $6.00. Helpful to teachers and students of physics, engineering, and allied subjects, and a "refresher" to those scientists who already make use of dimensional methods.

Electrical Breakdown of Gases. J. M. MEEK AND J. D. DRAGGS. 507 pp. Oxford University Press, New York, 1954. $10.50. Summarizes present knowledge about the mechanisms of growth of electrical discharges in gases and the transitions between different forms of discharges.

Essentials of Engineering Thermodynamics. HERMAN J. STOEVER. 279 pp. John Wiley and Sons, Inc., New York, 1953. $4.50. A complete, one-semester course for engineering students in an abridged version of *Engineering Thermodynamics*.

Experimental Metallurgy. A. U. SEYBOLT AND J. E. BURKE. 340 pp. John Wiley and Sons, Inc., New York, 1953. $7.00. Describes most of the important laboratory techniques which are now used in the preparation of metals and alloy specimens for further study.

First Year College Physics. CLARENCE E. BENNETT. 526 pp. The Ronald Press Company, New York, 1954. $6.00. Designed primarily for college freshmen. Its aim is to provide a firm grounding in classical physics.

Glossary of Nuclear Energy Terms. Section I. Physics (Formerly General Terms). National Research Council. 82 pp. The American Society of Mechanical Engineers, New York, 1953. $2.50. Definitions and explanations of physical terminology.

High Altitude Rocket Research. HOMER E. NEWELL, JR. 298 pp. Academic Press, Inc., New York, 1953. $7.50. The use of rockets for studying the ionosphere, earth's magnetic field, solar radiation, cosmic rays, and atmosphere.

History of the Theories of Aether and Electricity, 1900–1926. SIR EDMUND WHITTAKER. 319 pp. Philosophical Library, New York, 1953. $8.75. The revolution in physics which took place in the first quarter of the twentieth century.

Instrumental Analysis. JOHN H. HARLEY AND STEPHEN E. WIBERLEY. 440 pp. John Wiley and Sons, Inc., New York, 1954. $6.50. The operation of the major analytical instruments and the application of these instruments to research problems.

Interlingua at Sight. ALEXANDER GODE. 79 pp. Storm Publishers, New York, 1954. $2.00. It mediates between French, Spanish, Portuguese, Italian, English, and even German in that it embodies what these tongues have in common and leaves out what differentiates them.

Introduction to Concepts and Theories in Physical Science. Second Edition. GERALD HOLTON. 650 pp. Addison-Wesley Publishing Co., Inc., Cambridge, Massachusetts, 1953. Incorporating some of the conclusions drawn from teaching simultaneously for a number of years a

conventional type of introductory college physics course and also general education courses in physical science.

Introduction to the Theory of Error. YARDLEY BEERS. 65 pp. Addison-Wesley Publishing Co., Inc., Cambridge, Massachusetts, 1953. An important feature of advanced laboratory training.

Introductory Circuit Theory. ERNST A. GUILLEMIN. 550 pp. John Wiley and Sons, Inc., New York, 1953. Methods and concepts that are today's tools in advanced methods of network analysis and synthesis.

Leaders in American Science. ROBERT C. COOK. 703 pp. Who's Who in American Education, Inc., Nashville, Tennessee, 1953. $12.00. Biographical directory of eminent leaders in research, governmental, and educational scientific fields in the United States and Canada.

Linear Operators. RICHARD G. COOKE. 454 pp. St. Martin's Press, Inc., New York, 1954. $10.00. The present book gives merely those parts of the subject which specially interested the writer, and which at the same time appeared to fill a gap in the literature.

Magnetic Amplifiers. GEORGE M. ETTINGER. 88 pp. John Wiley and Sons, Inc., New York, 1953. $1.50. Intended to serve the practicing engineer or physicist who wishes to select the most suitable device for performing a given measuring or control function.

Magnetic Cooling. C. G. B. GARRETT. 110 pp. Harvard University Press and John Wiley and Sons, Inc., New York, 1954. $4.50. A balanced review of work done in England, Holland, and the United States.

Man, Rockets and Space. CAPTAIN BURR W. LEYSON. 188 pp. E. P. Dutton and Company, New York, 1954. $3.50. A clear, factual account of the latest developments and experiments by the U. S. Government on rockets.

Mathematical Methods for Scientists and Engineers. LLOYD P. SMITH. 453 pp. Prentice-Hall, Inc., New York, 1953. $10.00. Mathematical methods together with clear and understandable statements of the conditions which must be satisfied.

Matter, Energy, Mechanics. JAKOB MANDELKER. 73 pp. Philosophical Library, New York, 1954. $3.75. Unifies and extends relativity mechanics by introducing a new kinetic energy formula.

Methods of Theoretical Physics. Parts I and II. PHILIP M. MORSE AND HERMAN FESHBACH. Part I, 1037 pp.; Part II, 997 pp. McGraw-Hill Book Company, Inc., New York, 1953. $15.00 each. An extremely comprehensive and up-to-date treatment of the mathematical techniques used in theoretical physics.

Modern Developments in Fluid Dynamics High Speed Flow. Volume I. L. HOWARTH. 475 pp. Oxford University Press, New York, 1953. Deals with compressible flow at subsonic and supersonic speeds and includes both theoretical and experimental work.

Modern Developments In Fluid Dynamics High Speed Flow. Volume II. L. HOWARTH. 399 pp. Oxford University Press, New York, 1953. A companion volume to Goldstein's *Modern Developments in Fluid Dynamics*, in the same series.

Newton's Philosophy of Nature. H. S. THAYER AND JOHN HERMAN RANDALL, JR. 207 pp. Hafner Publishing Company, New York, 1953. $1.50. Selections from Newton's writings.

Nuclear Moments. NORMAN F. RAMSEY. 169 pp. John Wiley and Sons, Inc., New York, 1953. $5.00. A detailed examination of nuclear moment measurements as they affect the fields of nuclear physics, chemistry, and solid-state physics.

Physics for Medical Students. Third Edition. J. S. ROGERS. 405 pp. Melbourne University Press, New York: Cambridge University Press, 1954. $5.50. A book that should be in the hands of all teachers of medical students, as well as students themselves, and of qualified medical men.

Principles of Numerical Analysis. ALSTON S. HOUSEHOLDER. 274 pp. McGraw-Hill Book Company, Inc., New York, 1953. $6.00. Develops the mathematical principles upon which many computing methods are based and in the light of which they can be assessed.

Proceedings of a Conference on the Utilization of Scientific and Professional Manpower. National Manpower Council. Columbia University Press, New York, 1954. $3.50. Significant material of practical import for everyone concerned with the effective utilization of highly trained manpower.

Radio Receiver Design. Second Edition. K. R. STURLEY. 667 pp. John Wiley and Sons, Inc., New York, 1953. $10.00. Fundamentals of radio receiver design.

Recommendations for the Disposal of Carbon-14 Wastes. National Bureau of Standards Handbook 53. 14 pp. U. S. Government Printing Office, Washington, D. C., 1953. 15 cents. Best available opinions on the subject as of this date.

Roger Bacon. E. WESTACOTT. 140 pp. Philosophical Library, New York, $3.75. Life and legend of Roger Bacon.

Scientific Papers in Honor of Max Born. 94 pp. Hafner Publishing Company, New York, 1953. $2.50. Marks his retirement, after a tenure of seventeen years, from the Tait Chair of Natural Philosophy in the University of Edinburgh.

Simultaneous Linear Equations and the Determination of Eigenvalues. L. J. PAIGE AND OLGA TAUSSKY. 126 pp. National Bureau of Standards Applied Mathematics Series 29, Washington, D. C., 1953. $1.50. Invited reports from various mathematicians.

Snow Crystals. UKICHIRO NAKAYA. 510 pp. Harvard University Press, Cambridge, Massachusetts, 1954. $10.00. A record of factual, physical discovery, a gallery of beautiful structures and designs, a gallery of photographs of the crystals.

Soft Magnetic Materials for Telecomunications. C. E. RICHARDS AND A. C. LYNCH. 346 pp. Interscience Publishers, Inc., New York, 1953. $9.00. Papers presented at the Conference on Soft Magnetic Materials held at the Post Office Research Station, Dollis Hill, April, 1954.

Static Electrification. 104 pp. The Institute of Physics, London, 1953. 25s. Generation and effects of static electricity.

Statics and Strength of Materials. ROLAND H. TRATHEN. 506 pp. John Wiley and Sons, Inc., New York, 1954. $7.50. Principles of statics and strength of materials and general methods of applying them to engineering problems.

Tables of Coefficients for the Numerical Calculation of Laplace Transforms. HERBERT E. SALZER. 36 pp. National Bureau of Standards Applied Mathematics Series, Washington, D. C., 25 cents. Facilitate the numerical evaluation of infinite integrals expressible in the form of Laplace transforms.

Tables of 10^x. 543 pp. U. S. Government Printing Office. $3.50. An outgrowth of the program begun under the auspices of the Applied Mathematics Panel, National Defense Research Committee.

The Algebraic Theory of Spinors. CLAUDE C. CHEVALLEY. 131 pp. Columbia University Press, New York, 1954. $3.75. Oriented towards the algebraic and geometric applications of the theory of spinors.

The Collected Papers of Peter J. W. Debye. 700 pp. Interscience Publishers, Inc., New York, 1954. Papers have been of fundamental importance for many different branches of physics and physical chemistry.

The Cyclotron. W. B. MANN. 118 pp. John Wiley and Sons, Inc., New York, 1954. $2.00.

The Determination of Crystal Structures. H. LIPSON AND W. COCHRAN. 345 pp. The Macmillan Company, New York, 1954. $8.00. Crystal-structure determination from the stage at which a set of structure amplitudes has been obtained to the final accurate positioning of the atoms.

The Mechanism of Economic Systems. ARNOLD TUSTIN. 161 pp. Harvard University Press, Cambridge, Mass., 1953. Recent advances in our understanding of the mechanism of booms and slumps and economic fluctuations generally.

The Physics of the Stratosphere. R. M. GOODY. 187 pp. Cambridge University Press, Cambridge, 1954. $5.00. Concerns the physics of the atmosphere from above the weather-forming layers to the base of the ionosphere.

The Radio Amateur's Handbook. Thirty-first Edition. The American Radio Relay League. 800 pp. The Rumford Press, New Hampshire, 1954. $3.00. Written with the needs of the practical amateur constantly in mind.

Theory and Design of Electron Beams. Second Edition. J. R. PIERCE. 222 pp. D. Van Nostrand Company, Inc., New York, 1954. $4.50. Principles and problems most useful in understanding and in designing devices which use electron beams.

New Members of the Association

The following persons have been made members or junior members of the American Association of Physics Teachers since the publication of the preceding list [Am. J. Phys. 22, 350 (1954)].

Active Members

Baker, David Kenneth, Union College, Department of Physics, Schenectady, N. Y.

Barth-Wehrenalp, Gerhard, 1904-B Mather Way, Elkins Park, Pa.

Borrman, Bernice Bartlett, 718 5th Street, SW, Rochester, Minn.

Bosted, Nelson Paul, Apt. 25, Bucknell Village, Lewisburg, Pa.

Bufano, James, Jr., 902 Palisade Avenue, Union City, N. J.

Bulos, Fatin, American University of Beirut, Beirut, Lebanon

Burnett, Clyde R., Dept. of Physics, Pennsylvania State University, State College, Pa.

Cohn, George Irving, 60 E. 32 Street, Chicago 16, Ill.

Deloume, F. E., 2714 W. 154th Street, Gardena, Calif.

Drakeford, Foster Terry, J. C. Smith University, Charlotte, N. C.

Fried, Robert, The Polytechnic Institute of Brooklyn, 99 Livingston Street, Brooklyn 2, N. Y.

Gordon, Morton M., Dept. of Physics, 101 Benton, University of Florida, Gainesville, Fla.

Greene, Jack Bruce, 933 N. 34 Street, Milwaukee 8, Wis.

Hardy, Truly C., 66 Oakland Avenue, Port Washington, N. Y.

Harmon, Gerald Stearns, South Apt. 1-d, Orono, Maine

Hart, John Birdsall, 630 Blackburn Avenue, Hamilton, Ohio

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Lopez, Edward Thomas, 715 North Avenue, New Rochelle, N. Y.

Lundergan, Charles Donald, Saint Louis Univ., Parks College, East St. Louis, Ill.

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McManus, Edward Allen, Dept. Elect. Engr., U. S. Naval Academy, Annapolis, Md.

Montgomery, Donald J., Dept. of Physics, Michigan State College, East Lansing, Mich.

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Roser, Francis Xavier, Dept. of Physics, University of Santa Clara, Santa Clara, Calif.

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Socolar, Sidney Joseph, 5742 S. Drexel Avenue, Chicago 37, Ill.

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Wild, Robert Lee, Div. of Physical Sci. UCR, Riverside, Calif.

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Robinson, Donald Keith, Dept. of Physics, Dalhousie University, Halifax, N.S., Canada

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Tolmie, Ronald William, 436 Quinpool Road, Halifax, N.S., Canada

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Wood, John Herbert, 80 Marlborough, Boston 16, Mass.

Woods, David B., Dept. of Physics, Dalhousie University, Halifax, N.S., Canada

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Herbert Meredith Reese, 1873–1954

On May 10, 1954, PROFESSOR REESE died of a heart attack, bringing an end to the life of a man who had devoted nearly half a century to physics and the teaching of physics. Born December 1, 1873, in Baltimore, Maryland, he received the A.B. degree in 1897 and the Ph.D. in 1900 at Johns Hopkins University where he was a Fellow in physics in 1899–1900. From 1900 to 1903 he was on the staff at Lick Observatory and from 1903 to 1904 at Yerkes Observatory. He joined the staff of the Department of Physics at the University of Missouri in 1904 as Instructor, was promoted in 1907 to Assistant Professor, in 1912 to Associate Professor, in 1920 to Professor, and at retirement age he became Professor Emeritus in 1944 after forty years of active duty at Missouri, a period of service whose length exceeds that of any other physicist at this University. During the year 1911–12 he took leave from the University to study in Europe under several outstanding men including Planck and Lorentz.

Professor Reese was interested in spectroscopy and optics and while he contributed to these fields, his outstanding contribution was to the teaching of physics at all levels. He took great pleasure in setting up a well-equipped light laboratory and willingly spent much time to explain obscure points in this and other courses to students who displayed an interest in learning. Testimonials from undergraduate and graduate students and the writer's observations confirm the conclusion that Professor Reese was a most able teacher of physics. His major efforts were invariably directed toward getting his subject across to the students in a way that aroused their intellectual curiosity and nothing seemed to please him more than to have succeeded in clarifying a student's misconception. He published an intermediate-level textbook on *Light* in 1920, a *Laboratory Manual for General Physics* with Professor H. E. Hammond in 1934, eight papers on subjects of primary interest to physics teachers, three of a purely research character, and a few more notes or reports of various kinds. He served as chairman of the department for a number of years including many of the difficult depression years before the Second World War when the department had to be run with very inadequate support. He was a Fellow of the American Physical Society, a member of the Optical Society of America, Phi Beta Kappa, The Society of Sigma Xi, and Phi Mu Alpha.

Professor Reese was a man of high integrity, completely honest with himself and with others, and warmly sympathetic to the welfare of people around him. His artistic skill at the blackboard attracted attention and his rare sense of humor served not only to amuse his listeners, but often to smooth over otherwise difficult situations. Sincerity was one of his finest qualities and ostentatious display or bluff were not in his character. A sense of fair play pervaded his every decision.

A few years after retirement, Professor Reese, together with Mrs. Reese, moved to Georgetown, Colorado, where they resided until Professor Reese passed away. He is survived by his wife and by his daughter Mrs. A. W. Schultz of Clear Lake, Iowa. Though we mourn with them in the loss of this man, we nevertheless look back with pride on his many years of faithful service to physics and to the congenial association we had with him in his last few years of active service.—NEWELL S. GINGRICH.

RECENT MEETINGS

Oregon Section

The 66th meeting of the Oregon Section of AAPT was held at Reed College, Portland, Oregon, May 1, 1954. The following program of contributed papers was presented.

1. The dependence of magnetic forces on permeability. T. G. STINCHCOMB, *State College of Washington.*—It is well known that magnetic forces between magnets are inversely proportional to the permeability of the surrounding medium and that magnetic forces between current-carrying conductors are directly proportional to the permeability. Most textbooks in electricity and magnetism describe how magnetic materials are exactly equivalent to a distribution of currents (Ampére's theory of magnetism), yet few give the reasons for the difference in dependence of the forces on permeability. This difference is discussed. It arises from the fact that the surrounding medium cannot in general penetrate the region occupied by the magnetic material.

2. Optical absorption in silver chloride. F. C. BROWN, *Reed College.*

3. A piezoelectric experiment for the modern physics laboratory. J. J. BRADY, *Oregon State College.*—A single crystal holder is designed for use in studying both the direct and the converse piezoelectric effect. In the measurements of the direct effect a condenser is connected across the appropriate crystal faces and an electrometer tube used to measure the potential difference of the condenser terminals when the crystal is stressed. For the converse effect an optical system is used to measure the distortion of the crystal when a difference of potential is applied across the crystal faces.

4. Activities of the physics section at the U. S. Bureau of Mines Laboratory. D. M. MORTIMORE.—The facilities for instrumental analyses at the Northwest Electrodevelopment Laboratory were described with particular emphasis on the analysis of zirconium materials. The discussion included brief descriptions of analytical techniques employing modern instrumentation such as the Quantometer and the X-ray Spectrograph. A unique application of the Quantometer has been the direct spectrographic analysis of 500-pound zirconium ingots using a special holder to permit a rapid analysis of any point on the surface of the ingot.

5. An extension of Bloch's theorem in quantum mechanics. J. F. DELORD, *Reed College.*

Thereafter, several McGraw-Hill—AAPT short films on topics in physics were shown.

The following schedule of future meetings was approved. Fall meeting: *U. S. Bureau of Mines Laboratory*, Albany, Oregon, Friday afternoon and evening, November 5, 1954. Winter meeting: *Portland State College*, Portland, Oregon, Saturday, February 19, 1955. Spring Meeting: *Washington*

State College, Pullman, Washington, Saturday, May 7, 1955.

The nominating committee presented the following slate of officers which was then elected by a unanimous ballot: President, Dr. Francis Dart, *University of Oregon;* Secretary, Dr. Duis Bolinger, *Oregon State College;* Historian, Brother Godfrey Vassallo, *University of Portland.* Designation of the representative of the section to the meeting of the national Council of the AAPT was left as business of the fall meeting when it would be more definitely known what members expected to attend the New York meeting.

KENNETH E. DAVIS
Secretary

Southeastern Section, American Physical Society

The Twentieth Annual Meeting of the Southeastern Section of the American Physical Society was held on April 1, 2, and 3, 1954, at the University of Tennessee in Knoxville and included in its program three invited and seven contributed papers concerned primarily with the teaching of physics. The invited papers were:

Some remarks on general education science courses. R. T. LAGEMANN, *Vanderbilt University.*

Uses and abuses of demonstration experiments in teaching physics. R. M. SUTTON, *Haverford College.*

General physics laboratory without written directions. GUY FORMAN, *Vanderbilt University.*

A. E. Ruark of the University of Alabama spoke at the dinner meeting on **Physics in the Southland, 1934–1954.** The abstracts of the contributed papers appear below. The research papers, constituting the remainder of the program, are reported in the *Physical Review.*

1. Physicians as physicists. E. SCOTT BARR, *University of Alabama.*—The importance of the contributions of physicists to the science of medicine is recognized by all physicists. However, a realization that physicians have made notable contributions to the science of physics is not so common. A survey of some 600 physicists who made contributions prior to 1900 has shown that about 40 contributors are identifiable as practicing physicians, and it seems probable that there are others among the group for which biographical information was not obtained. Some of the contributions of these men are: the critical point studies of Andrews, the discovery of double refraction by Bartholinus, the fundamental work in heat by Black, Cassegrain's reflecting telescope, Charles' law for gases, D'Arsonval's galvanometer, Gilbert's fundamental studies of magnetism, Mariott's gas law (as Boyle's law is known on the continent), Mayer's principle of conservation of energy, Poiseuille's work on viscosity, Young's modulus of elasticity, etc. Those who teach physics to premedical students may find a mention of such men desirable during the progress of the course work, and those who teach physicists and engineers may find it worth while to indicate to their students this inter-relation of scientific effort.

2. The *plus* values in physics teaching. RONALD A. McGEE, *Southern State College, Magnolia, Arkansas.*—Most physics teachers find that one of the largest problems in teaching general physics is to secure and hold the interest of the students. The writer has met this problem by using such hobbies as amateur radio and photography to build interest in physics. Even the student who is taking the course for credit only will make an effort to learn physics when he sees that he can use it in his hobby. At first the students were given a lecture on the subject, "Amateur Radio," in which some of the applications of physics were pointed out. Then the students worked on projects in building crystal sets, 4- and 5-tube receivers, and transmitters; and studied handbooks on amateur radio operation and construction in amateur radio clubs. Photography and other hobbies were used in a similar manner.

3. The AEC radiological physics fellowship program. E. E. ANDERSON, *Oak Ridge National Laboratory.*—Radiological Physics, often termed Health Physics, deals with the techniques and basic physics involved in radiation detection, measurement, and shielding, particularly as they apply to the protection of personnel from harmful exposure to radiation. The program of the U. S. Atomic Energy Commission (AEC) and the expanding use of radioisotopes in industrial production, research, and medicine have created an increasing demand for radiological physicists. The AEC, public health agencies, the military establishment, industry, universities, and hospitals all need people trained in radiation monitoring, and in the handling, measurement, and disposal of radioactive materials and the associated by-products. The Fellows spend the academic year taking formal courses at the university to which they are assigned and then transfer to the corresponding AEC installation where they work approximately three months in Applied Health Physics. Fellows whose work has demonstrated unusual capability may be eligible for extensions of their fellowships to complete the qualifications for a master's degree. Job opportunities in the field continue to increase in both number and variety.

4. Estimation of numbers of physicists in training during the next decade. MARSH W. WHITE, *The Pennsylvania State University.*—Reasonably reliable data are presented to show the basis for estimates of the numbers of physicists in training at various levels during the next ten years. Bachelors degree physicists were produced at rapidly increasing numbers after World War II, reached a peak of 3800 in 1949–1950, and declined to a constant rate of about 2400 which will continue to about 1957–1958, after which the number will again increase. The number of Master's degrees awarded in physics exceeded 1000 in 1949–1950, but has now fallen off to about 800, which number will remain nearly constant until about 1957–1958. The rise thereafter will parallel the increase in undergraduate degrees awarded in physics. Doctorate degrees awarded to physicists rose greatly from a low of less than 50 in 1945 to an all-time high of 500 in 1952–1953. The decline since then indicates a fairly level production of

about 400 per year until the increased enrollments should lead to a steady upturn beginning about 1961–1962. All of the data from recent studies indicate a continuing demand for physicists. It seems probable that there will be severe shortages of physicists during the foreseeable future.

5. The solution of problems by fundamental relationships. JOSEPH H. HOWEY, *Georgia Institute of Technology.* —In teaching general college physics we are concerned with the excessive amount of material to be covered, and with the most effective treatment of the material selected for presentation. In this connection, several ideas gained from classroom experience are submitted. These include a way of applying the fundamental wave equation, $v = f\lambda$, so that this one equation will handle all problems where the Doppler effect is involved. This is accomplished by defining v as the magnitude of the velocity of the wave train relative to the thing having the frequency f, and remembering that the wavelength of a wave train does not change as it moves along. The same equation then applies either to the source or hearer, giving $v_1/f_1 = \lambda = v_2/f_2$. This viewpoint leads to the solution of problems by a repeated application of one basic relationship, rather than by merely substituting values in a derived equation of limited application which is circumscribed by arbitrary rules for algebraic signs.

6. A model for demonstrating the motion of ions in a cyclotron. F. T. HOWARD, *Oak Ridge National Laboratory* (introduced by Robert S. Livingston).—The construction and operation of a simple device for demonstrating the motion of ions in a fixed frequency cyclotron are described. The device consists essentially of two photographic negatives. In one the spiral path of ions is traced from the ion source to the internal target. In the other the radial motion of ions is shown independent of orbital motion. When these two negatives are superimposed before a light source and are rotated, one with respect to the other, ions appear to originate in the ion source, traverse their calculated orbits, and strike the target. The phenomena of ion bunching, phase shift due to rf tuning, and relativistic phase lag can be clearly shown.

7. Electron optics at Fisk University. GERTRUDE F. REMPFER, BENJAMIN F. PEERY, HOWARD J. FOSTER, AND JESSE L. REEVES, *Fisk University* (introduced by Nelson Fuson).—A description will be given of the installation at the Physics Department of Fisk University of an electron optical system applicable to electron microscopy and electron diffraction. The instrument, designed by one of us (G. F. R.), uses electron optics in an assembly which was built in a small physics department machine shop. A number of slides will be shown illustrating the electron optical system and its performance. This type of instrumentation can be used for both electron optical and ion optical applications. It is currently being used in an introductory course in electron microscopy at Fisk University.

At its meeting, which was attended by 270 physicists from the area, the Section elected W. M. Nielsen of Duke University as Chairman for 1954–1955. Other officers are: Vice-Chairman, M. S. McCay, the *University of Chattanooga;* Secretary, Dixon Callihan, ORNL; Treasurer, R. T. Lagemann, *Vanderbilt University;* Member of Executive Committee, R. C. Williamson, *University of Florida.* The 1955 Meeting will be held at the University of Florida on April 7, 8, and 9.

DIXON CALLIHAN
Secretary

Oregon Section

The business session of the 65th meeting of the Oregon Section was called to order by the Section President, Fred W. Decker, Oregon State College, at the University of Oregon on February 20, 1954. Discussion of the AAPT Council meeting with respect to the subject of dues resulted in unanimously instructing the Secretary to communicate the following sentiments to the Executive Committee:

"We wish to express strongly our conviction that any increase in the membership dues of the AAPT at present would be both unnecessary and ill advised.

We fail to see the logic of comparing our organization with a company or an industry in accumulating a large "emergency" fund (which, however, is not being used in an emergency). We feel that the present situation is *not* a serious crisis and that even with present trends such a crisis should not exist for about another six or seven years.

With the present membership campaign already showing signs of halting the previous downward trend in membership it would be more reasonable to try to establish the organization on an even keel financially through increased membership than by increasing the burden on the present members.

It is our considered opinion that an increase of $6 to $8 (as presently contemplated) will certainly result in a loss of members—both present and potential. We object to any such increase at this time."

The Business session adjourned with a rising vote of thanks to the host institution, the University of Oregon.

The following papers were presented:

1. The origin of the magnetic field of the earth. G. D. HOYT, *University of Oregon.*—The core of the earth is presumed to be a metallic conductor. The positive ions and conduction electrons of the conductor are described separately by wave functions which for the positive ions are localized at lattice points and for the conduction electrons are (ideally) spread out uniformly throughout the core at 0°K like free electron waves. At elevated temperatures, the ions scatter the free electron waves, disturbing the uniformity of the electron distribution. Describing the ensemble of electrons as partly free waves and partly point charges, the ions as point charges, and superimposing the earth's rotational velocity and thermal velocities, an elementary calculation shows that the value and direction of the magnetic field at the center of the earth will depend on the fraction of electrons describable

as point charges. This fraction, hence the field, is then temperature dependent. Since the correct description of the electronic wave functions depends on the interaction with the positive ion lattice, there is evidently no thermodynamical reason for choosing a description which would give a zero field to a massive rotating metallic conductor at an arbitrary temperature.

2. Observation of short period fluctuations in the geomagnetic field. PHILIP A. GOLDBERG, *University of Oregon.*—The results of an investigation for extending the known picture of geomagnetic variations to frequencies above 1 cy/sec are presented. To detect the fluctuations a detector coil of 1 m diameter and 3500 m^2 effective area was used. The coil signals were amplified with an amplifier of 2.2 to 78 cy/sec band pass, and 0.2 μv input noise, for registration on a continuously recording thermal power meter. The system was capable of detecting signals as weak as 3×10^{-7} gauss/sec $= 3 \times 10^{-11}$ w/m^2/sec rms. The observations were made in isolated locales (to insure freedom from 60 cy/sec interference) in Southern California. The detector coil was buried upright to its full depth, in rock and in sand, with the plane of its turns normal to geomagnetic meridian. Magnetic fluctuations were observed occurring in bursts of duration usually less than 1 sec. On an active day bursts of rms value 6×10^{-10} w/m^2/sec occurred as often as 5/min. On a quiet day bursts were found to occur as infrequently as every 5 min with values generally less than 10^{-10} w/m^2/sec. Mechanisms capable of causing the burst events, such as fluctuations in ionospheric currents, were discussed.

3. New technique for evaluation of cloud seeding operations, R. L. LINCOLN, *Oregon State College.*—No completely satisfactory technique for evaluating cloud seeding operations for the purpose of increasing precipitation has yet been devised. Several methods used in the past by impartial evaluators were reviewed and their weaknesses are cited. An improved technique devised at Oregon State College by a team of three meteorologists combines weather typing and a multiple regression analysis in which several meteorological variables are used on a daily basis. This appears to be an important step in the evolution of evaluation technique to sensibly detect smaller and smaller possible increases due to seeding.

4. The complex stellar system Algol. E. G. EBBIG-HAUSEN, *University of Oregon.*—The complex stellar system of Algol is now known to consist of at least five components. Only one of these stars (possibly two) are bright enough to be recorded spectroscopically. The multiplicity of the system is revealed through the secular change in the velocity of the center of mass of the close system whose period is about 2.9 days and also through the variations in the times of eclipse of that system. The nature of the spectrum and its anomalous behavior is of considerable astrophysical interest.

5. Report on New York meetings of the APS and the AAPT. R. T. ELLICKSON, *University of Oregon.*

6. Positronium. B. CRASEMAN, *University of Oregon.*—A positronium atom is formed when a negative electron is captured into a Bohr orbit about a positron, or vice versa. Ortho-positronium has a mean life of 1.5×10^{-7} sec and decays by three-photon annihilation; para-positronium decays into two photons with a mean life of 1.2×10^{-10} sec. In a brief review of the subject, the theoretical predictions of Pirenne,[1] Wheeler,[2] and Ore and Powell[3] are outlined. The experimental proof of the existence of three-photon annihilation by Rich[4] and by DeBenedetti and Siegel[5] is described, and Deutsch's measurements of the half-life of ortho-positronium are sketched.[6] The determination of the fine structure splitting of the ground state of positronium through a measurement of the quenching of three-quantum decay in a magnetic field is described.[7,8]

[1] J. Pirenne, thesis, University of Paris (1944).
[2] J. A. Wheeler, Ann. New York Acad. Sci. **48**, 219 (1946).
[3] A. Ore and J. L. Powell, Phys. Rev. **75**, 1696 (1949).
[4] J. A. Rich, Phys. Rev. **81**, 140 (1951).
[5] S. DeBenedetti and R. Siegel, Phys. Rev. **85**, 371 (1952).
[6] M. Deutsch, Phys. Rev. **83**, 866 (1951).
[7] M. Deutsch and S. C. Brown, Phys. Rev. **85**, 1047 (1952).
[8] J. Wheatley and D. Halliday, Phys. Rev. **88**, 424 (1952).

7. Some electronic properties of the silver halides. F. C. BROWN, *Reed College.*—The drift mobility and mean range of electrons and holes at liquid nitrogen temperatures have been investigated in AgCl. The experimental techniques are discussed and results given. Single, well-annealed samples are prepared from crystals grown by the Bridgman technique. They are operated as crystal counters detecting the ionization produced by individual beta particles from a small beta-ray spectrometer. Conduction pulses are amplified by equipment which is fast so that pulse shapes as well as pulse heights can be analyzed. Their shape agrees with theory for uniform trapping of conduction electrons in the volume of the crystal and can be shown to readily yield values for the mean range w. Examples of pulses are given for a crystal operated at high electric field strength (nearly saturated conditions) and at low field (well below saturation). The results at $85 \pm 5°K$ for a total of nine separate runs on five different crystals yield values of electron mobility in the range 250 to 300 cm^2/volt-sec. The best value of energy per ion pair for 0.62 Mev beta particles is 7.5 ± 0.5 electron volts in agreement with the work of Van Heerden on AgCl. For the samples tested little or no conduction can be attributed to holes at these temperatures. It is planned to study some of the electronic properties of crystals containing added impurities. The work has been supported by the National Science Foundation and the Research Corporation.

8. Some characteristics of a ground germanium surface. P. CAMP, *Reed College.*—An important feature of the germanium crystals used in semiconducting devices is the surface condition. For this reason, it is desirable to know the extent to which a crystal surface is damaged by processing (cutting and grinding). This disturbed layer is generally removed by etching. Estimates of the thickness of this layer have been made by x-ray techniques and by direct hole lifetime measurements. A simple method of obtaining a somewhat more precise measure-

ment will be described. It is found that the depth is orientation dependent. The experiment also gives some information on the nature of the damage and the mechanism of etching.

9. Laboratory experiments for nontechnical students. F. E. DART, *University of Oregon.*—The laboratory objectives and requirements of an elementary science course intended for students who will not take any further work in science were considered. The practicability of eliminating preoccupation with experimental techniques and complicated apparatus while retaining the essential physical concepts of an experiment were illustrated by a few selected experiments.

10. New methods and techniques in shop and instrumentation. O. J. FILZ, *Oregon State College.*—Summary of a series of seminar talks titled "New Methods and Techniques in Shop and Instrumentation." This series of seminar talks is designed to bring to the attention of the physics teachers, graduate students, and research personnel new materials and techniques, and their application in the construction of laboratory apparatus, thesis projects, and research equipment.

KENNETH E. DAVIS
Secretary

Central Pennsylvania Section

The Central Pennsylvania Section of the American Association of Physics Teachers met at Juniata College on April 9–10, 1954. This was the second meeting for the Section.

Seven contributed student papers and five invited papers comprised the program.

Invited Papers

1. The mighty midget. EUGENE D. LAVERY, Supervisor of Customer Information, *The Bell Telephone Company of Pennsylvania.*

2. Analysis of Cassegrainian type telescopic systems. PAUL R. YODER, JR., *Frankfort Arsenal.*—Recent work at Frankfort Arsenal has shown the optical effect of modifying the Cassegrainian type telescope objective in order to simplify manufacture. Various optical tolerances are applied to determine the conditions under which this modified optical system can be used in an astronomical telescope of moderate aperture and focal length.

3. Motion picture of a diffusion-type cloud chamber. JOHN J. HEILEMANN, *Ursinus College.*—A diffusion cloud chamber was described in which the tracks of alpha and beta particles are illuminated with sufficient brightness to make 16-frame-per-second motion pictures possible. On the 200-foot film were pictures of an empty chamber with tracks from contamination or cosmic-ray particles as well as the Compton electrons resulting from the gamma rays from a luminous instrument dial held near. The film included pictures of a chamber containing a speck of radium salt, showing the limited range of the alpha

particles, the longer and lighter tracks of the beta particles, and the short curved paths of the secondary electrons resulting from the action of the gamma radiation on the material of the chamber. A chamber containing a source of polonium in which practically the entire activity is due to alpha particles of a shorter range than those from the radium sample was also shown.

It was pointed out that short loops could be made of any part of the film which seems to contain interesting events. These loops are also useful for demonstration purposes.

4. Basic physical concepts of nuclear magnetic resonance. E. L. HAHN, *Watson Scientific Laboratory, Columbia University.*—The recent technique of nuclear magnetic resonance, pioneered by Bloch at Stanford and Purcell at Harvard, is now very much the vogue in studies of the solid state and chemical analysis. The atomic nucleus for many of the elements possesses the property of spin angular momentum with an accompanying magnetic moment. The nuclear moment precesses with a prescribed Larmor frequency in a magnetic field, very much in the sense that a spinning mechanical top precesses with a prescribed frequency due to the torque exerted by the earth's gravitational field. A model of the nuclear resonance is seen by tying a string to a "sleeping" mechanical top (spinning in vertical alignment with gravity) and pulling the string in such a direction that the string tension is tangent to the circle of natural precession of the top as it deviates from the vertical. The top will gradually nutate toward the plane perpendicular to the vertical, and the precessing component of spin angular moment increases in this plane. Similarly a "sleeping" ensemble of nuclear spins, aligned by a large magnetic field H_0, will be acted upon by the effect of a small radio-frequency field H_1, now replacing the string, which precesses in a plane perpendicular to H_0 at the natural frequency of Larmor precession. The "sleeping" ensemble of spins constitutes a macroscopic magnetic moment M_0 aligned by H_0. As the component of M_0 increases in the plane perpendicular to H_0, a pickup coil has a voltage induced in it at the Larmor frequency due to dipole field lines cutting the coil. The measured values of Larmor frequencies and the shape of the voltage of resonance response gives valuable information about the character of local magnetic fields in many substances.

5. The use of radioactive materials in biology and medicine. G. L. BROWNELL, *Massachusetts General Hospital.*—Positron-emitting isotopes offer possibilities of better resolution for tumor localization than is attainable with single-detector probe or external measurements of gamma emitters.

The use of P^{32} combined with the Robinson-type G-M probe counter has proved to be of value during surgery of intracranial tumors. The investigation has covered two aspects: the development of apparatus for detection of annihilation radiation from positrons, and a biological investigation of the relative concentration of various isotopes in tumor and normal brain tissues.

Contributed Papers

6. Detection of minute amounts of oil in water. ROYAL P. FISHER, *Pennsylvania State University.*—A search of the literature covering the period 1911 through 1952 reveals that oils may be detected by color, fluorescence, luminescence, spectroscopic analysis, and surface tension. The true color of petroleum oils may be determined by colorimeter, a filter-photocell instrument, or a grating-photocell instrument.

Reports indicate that one part of oil in 100 000 can be detected fluoroscopically. Vitamin removal may cause luminescence in refined oils. Crude oils, however, usually lack luminescence. Luminescence may be produced by supersonic waves in water containing dissolved air.

Accurate estimations of traces of petroleum in water of the order of 0.2 parts per million to 5 parts per million allegedly may be calculated by measuring the decrease in surface tension of the water when it becomes covered with an oil film.

Our experimentation shows correlation between surface tension and the quantity of oil in water. The sensitivity of this method is discouragingly low, however.

Details are given in the literature of the techniques used in detecting oil by ultraviolet light. A "Fluorograph" is reported capable of measuring concentrations as low as one part in a billion. However, it is necessary to use an infrared spectrophotometer for the analysis of most hydrocarbons, because saturated hydrocarbons do *not* absorb in the ultraviolet region.

7. The separation of certain gases from water. E. A. LUSTER, *Pennsylvania State University.*—In analyzing for nitrogen and oxygen dissolved in water, it is essential that the atmosphere be excluded from the time of collection of the sample until the analysis is complete. A method of collecting water samples and sealing them in glass tubes to preserve them against change in gas content or nature due to changes in temperature or external pressure is described.

The samples are introduced into the analytical system without subjecting them to the atmosphere by breaking the tubes within the system, and finally, a thermal method utilizing heat and liquid air to extract the sample and free the gases from water vapor, carbon dioxide, and other interfering gases is discussed. The oxygen and nitrogen content are then measured by reactions with phosphorus and lithium, respectively.

8. An experiment to measure the half-life of an isotope. LAIRD D. SCHEARER, *Muhlenberg College.*—The purpose of this work was to prepare an experiment to measure the half-life of the isotope, ThB, suitable for an advanced undergraduate physics course. The method followed is similar to the one described by Robinson and Cole [Am. J. Phys. **21**, 469 (1953)].

A $5N$ solution of thorium nitrate is electrolyzed for 30 minutes and ThB and ThC are deposited on the cathode. A scintillation counter employing a 931 A photomultiplier tube in combination with a scaling unit is used to follow the alpha activity of ThC′, a product of the ThB disintegration. Because of the extremely short half-life of ThC′, secular equilibrium is reached quickly and the radioactivity is indicative of the ThC present.

The activity of the cathode deposit is plotted as a function of time over a period of 45 hours. The latter portion of the curve is indicative of the longer-lived substance, ThB in this case. The curve at short times represents the activity of the ThC deposited on the cathode plus the activity of the ThC which is formed by the disintegration of the ThB. A subtraction of the ThC formed by the ThB from the total activity leaves a curve representing the activity due to the initial deposit of ThC. Analysis of the curves gives results of 10.7 hours and 1.1 hours for the half-lives of ThB and ThC, respectively.

9. Investigation of the variation of beta- and gamma-ray absorption coefficients with magnetic fields. ANDREW KONRADI, *Franklin and Marshall College.*—Investigation was made of any change in the coefficient of absorption of permalloy in a magnetic field as the strength and the direction of the field were varied. Electrons of 2.32 Mev energy from a $_{91}Pa^{234}(UX_2)$ source were used in this investigation. The particles were detected with a Geiger counter. No change in the coefficient was observed.

10. Helium content indicates age of meteorites. E. DALE MCGARRY, *Pennsylvania State University.*—Dr. F. A. Paneth of the University of Durham, England, derived the age of fifty meteorites from the fact that helium is produced by the decay of uranium and thorium existing in metallic meteorites. He assumed there were no other helium producing processes seriously affecting this content.

Dr. Carl A. Bauer, now at The Pennsylvania State University, pointed out the strong correlation between the age and size of the meteorites, and has shown by his work that cosmic radiation is responsible for an appreciable amount of this helium. This paper consists of an account of Dr. Bauer's work, and a discussion of the helium-content method of measurement.

11. Interference spectroscopy. E. RUTH MCCRACKEN, *Wilson College.*—A study of the interference patterns for different sources of light was made by using a Hilger Fabry-Perot etalon in combination with a Bausch and Lomb Constant Deviation Spectrometer. Sources of light included the sodium vapor lamp, mercury discharge tube, and iron arc. Photographs were taken of the interference patterns thus obtained. These patterns were measured on a Gaertner Comparator which measures to 0.001 mm.

12. Microwave spectral line breadths.* PAUL M. MOSER, *University of Delaware.*—The origin and shape of microwave spectral lines produced by rotational energy transitions in linear molecules are discussed briefly. A microwave spectrograph, designed for studies of the pressure and temperature dependence of microwave absorption line widths, is described. Experimental results on the linear molecule $O=C=S$ are given.

* Supported by the U. S. Air Force Office of Scientific Research.

At this meeting the following officers were elected. President: Professor Paul Yoder, *Juniata College;* Vice-President: Professor Dorothy W. Weeks, *Wilson College;* Secretary-Treasurer: Dr. Robert A. Boyer, *Muhlenberg College;* Representative on AAPT Council: Professor Kenneth V. Manning, *The Pennsylvania State University.*

The next annual meeting is scheduled at Wilson College, April 1–2, 1955.

DOROTHY W. WEEKS
Vice-President

Illinois Section

Dr. G. M. Almy, Associate Head of the Department of Physics at the University of Illinois, invited the Illinois Section of the American Association of Physics Teachers to hold its annual fall meeting on the Urbana campus Friday evening and Saturday, October 9 and 10, 1953, with Professor R. F. Paton of the University of Illinois acting as local chairman in charge of the program and arrangements. The group met for dinner at the University Men's Club on the Urbana campus at 6:15 p.m. This was followed by an evening lecture on "Plastic Deformation of Crystalline Solids," given by Professor J. S. Koehler. The Saturday morning program consisted of three one-hour lectures: "Nuclear Magnetic Resonance," by Professor C. P. Slichter; "Recent Research in Semi-Conductors," by Professor T. A. Murrell; "New Nuclear Particles," by Professor G. F. Chew. This was followed by a visit to the laboratory of Professor Slichter to observe his experimental method of studying nuclear magnetic resonance.

The section met for luncheon in the Colonial Room in the Illini Union Building. The business meeting was held immediately following the luncheon. The afternoon was given over to planned visits to the laboratories housing the cyclotron, betatron, low-temperature equipment, solid-state physics, transistor research, computing machines, and magnetic resonance work. The entire program was deeply appreciated by the members of the Illinois

Section. The four lectures given were excellent summaries of the work in the various fields and were indeed well presented. The opportunity for the planned conducted trip through the various laboratories was of great value and was very effective in supplementing the forenoon program. The executive committee looks forward to meetings similar to the one held at the University of Illinois in the fall of 1954 where firsthand information and observation of the most recent equipment being used in the various fields can be had. The section is making plans for the presentation of a series of ten-minute papers at its annual meeting every other year, or more often as seems desirable.

The new constitution of the Illinois Section, patterned after the national constitution, is functioning well and is particularly effective in promoting more breadth and continuity in programing and in developing widespread interest among the teachers of physics throughout the state.

The organization is actively promoting close cooperation between the schools of engineering and the college physics teachers in setting up pre-engineering courses in keeping with the increased physics requirement demanded by many engineering schools. The Illinois Section offered to cooperate with the ASEE in a symposium on physics for engineers at the spring, 1954, meeting of the ASEE to be held at the University of Illinois.

The membership of the association is constantly increasing and now stands at 92 members. The junior membership in the State Association and the membership in the National Association of the AAPT by Illinois residents are also increasing. The Illinois Section officers and the Executive Committee are anxious to cooperate with the National Association in all ways possible and are actively engaged in increasing the membership of the National Association.

GLENN Q. LEFLER
Section Representative

Proceedings of the American Association of Physics Teachers

Twenty-Third Annual Meeting, January 28–30, 1954

THE twenty-third annual meeting of the American Association of Physics Teachers was held in New York City, January 28, 29, and 30, 1954, with Columbia University, Barnard College, and the City College of New York acting as hosts to the Association. The American Physical Society held meetings at the same time, Columbia University being the host institution. The registration of the members of both the Association and the Society took place in Pupin Laboratory, starting at 9:00 A.M. January 28. The sessions of the Association were held in

Casa Italiana during the first day of the meetings. The first morning program, with Professor Henry A. Boorse presiding, consisted of five contributed papers, each dealing with some phase of the subject of physics and its classroom presentation. The panel discussion "Research Subsidies and College Teaching" presided over by Professor Karl S. Van Dyke, Wesleyan University was carried on with significant interest to all members. The panel consisted of Dean Herbert E. Longnecker, Graduate School, *University of Pittsburgh*, Professor Theodore

Soller, *Amherst College*, Professor Walter C. Michels, *Bryn Mawr College*, and Doctor Charles H. Schauer of the *Research Corporation*. Various facets of problems involved in combining teaching and research in the colleges were ably presented by the panel members.

Old friends chatted together and met new friends in the cafeteria dining room of Hewitt Hall, Barnard College, the lunchtime headquarters for members of the Association.

At 2:00 P.M., in the Casa Italiana, the second panel discussion, "The Responsibility of the Physics Teacher in Engineering Education" was presided over by Professor Joseph H. Keenan, *Massachusetts Institute of Technology*. Members of the panel at this session were, Dean L. E. Grinter, *University of Florida* and president of the American Association for Engineering Education, Professor Nathaniel H. Frank, *Massachusetts Institute of Technology*, Professor L. P. Smith, *Cornell University*, and Dean Homer L. Dodge, *Norwich University*. Dean Grinter presented the point of view of those institutions which have become dissatisfied with the physics taught for engineers in the physics departments.

The second afternoon session consisted of eight contributed papers, Professor R. Ronald Palmer, *Beloit College*, presiding. These papers presenting a variety of laboratory experiments were well received by the members of the Association who filled the auditorium of the Casa Italiana.

The Friday morning session of the Association was held at the *City College of New York* in the Townsend Harris Auditorium. Professor Vernet E. Eaton, *Wesleyan University*, was the moderator of a very interesting and historic session on the topic "The Functions and Mission of AAPT." Dean Harold K. Schilling reviewed the history of the Association and pointed out some of the areas which have as yet been undeveloped by the Association. His frank appraisal was met with frank and challenging queries. The place and function of the high school teacher of physics in the Association was suggested as one topic for further study. Dean Thomas H. Osgood, *Michigan State College*, editor of the *Journal*, presented the topic, "What goes into the *American Journal of Physics*." The material included is selected as to its (1) use and interest

to a physics teacher whether on graduate or undergraduate level; (2) interest as a review article; (3) professional interest to physicists whether teachers or research workers in industry or government.

One of the highlights of an excellent program was the invited paper presented by Doctor C. N. Hoyler of *Radio Corporation of America*, "A Demonstration of Color Television," Professor Henry A. Boorse, *Barnard College*, presiding. Following a short historical sketch of the efforts to transmit a picture, the most recent developments of the R.C.A. method of compatible transmission of a color picture were presented. The results of the method which suitably transmits a signal which is coded with the necessary information were demonstrated by "transmitting" several kodachrome slides from a "camera" to an experimental model of a color television receiver.

The Friday afternoon joint ceremonial session crowded the McMillin Theatre of *Columbia University*. Professor Fermi, in presenting the address of the retiring president of the American Physical Society, answered with wit and humor the question, "What Can We Expect to Learn from High Energy Accelerators." The relationships between the "strange particles" and their role in the structure of matter as well as the character of the nucleus are problems which need more study with higher energies.

The Oersted Medal was awarded to Professor C. N. Wall of the *University of Minnesota*. In presenting the medal Professor Mark Zemansky of the Committee on Awards reviewed the training of the recipient, beginning with *Stivers High School*, Dayton, Ohio, continuing through *North Central College*, Naperville, Illinois, and completing his graduate work at the *University of Illinois*. Professor Zemansky paid tribute to the ability of Professor Wall as a teacher and as one who was able in sending on to graduate school many of his students, while teaching at *North Central College*.

Professor Wall, in accepting the award, presented his address, "The Metaphysics of a Physics Teacher," a dissertation dealing with his philosophy of teaching.

The twelfth Richtmyer Memorial Lecture was presented by Professor John A. Wheeler,

Princeton University, on the topic, "Fields and Particles." That evening at seven o'clock in the ballroom of the Hotel New Yorker the members of the American Physical Society and the members of the American Association of Physics Teachers joined in the annual banquet. Doctor P. E. Klopsteg introduced the officers of the two organizations and their wives. Doctor Enrico Fermi introduced the speakers of the evening. Doctor S. A. Korff, *New York University*, related his most interesting experience in setting up and maintaining a laboratory at high altitude in northern Alaska. Doctor Leon Brillouin, IBM, Watson Laboratory, recounted early experiences with his friend Professor Lorentz. President Grayson Kirk, *Columbia University*, gave the address of the evening. At this banquet the second Oliver Ellsworth Buckley Solid State Physics Prize was presented to Doctor John Bardeen, *University of Illinois*.

Saturday morning the Association held their meeting in the Harkness Theatre, *Columbia University*, Professor Reginald J. Stephensen, *College of Wooster*, presiding. Six contributed papers dealt with various phases of the philosophy of physics and the responsibilities of the physics teachers to society. Two invited papers were presented in the next session with Professor T. D. Phillips, *Marietta College*, presiding. Professor J. W. Buchta, *University of Minnesota*, in his paper, "The Summer Institute for Teachers of Physics," spoke regarding the experiences at the Institute during the past summer. The experience with both college physics teachers and high school physics teachers, should help to an understanding of the teaching problems on these two levels. The plans which seem probable at this date were related. It would certainly be to the advantage of the physicists if such an institute could be held each summer. Doctor Bowen C. Dees, Program Director for Fellowships, National Science Foundation, spoke on the topic, "Fellowship Program of the National Science Foundation."

At the Annual Business Meeting it was voted to confer an honorary membership of the Association upon Doctor Detlev Bronk, Former President of *Johns Hopkins University*, President of the National Research Council, and Head of the Rockefeller Institute for Medical Research.

A change in the By-Laws was approved which will permit the increase of dues to some figure not to exceed $8.00 per year. A recommendation made by the Wisconsin Section for the amending of Article IX of the Constitution was passed. Proposed amendments may be originated from the membership by petition or by the Council. Recognition was made of those members who had passed on in death this past year. Professor Winans, *University of Wisconsin*, announced that tape recordings of the various programs were available. Action was taken to set up a committee to study the problems of the high school teachers. Professor White suggested that the high school teachers be invited to the sectional meetings. The new officers for 1954 were announced as follows: Professor Marsh White, *Pennsylvania State College*, president; Professor R. Ronald Palmer, *Beloit College*, president-elect; Professor R. F. Paton, *University of Illinois*, secretary; Professor F. W. Sears, *Massachusetts Institute of Technology*, treasurer.

At the afternoon session the Audio Visual Aids Committee made a report and showed several films which had been prepared under their direction. Two films, of about ten minutes showing time each, were on the subject of Radioactivity and Nuclear Fission. Three additional films, of about the same showing time, presented the topics of Simple Harmonic Motion, Transverse Standing Waves, and Longitudinal Wave Propagation.

The final session, James M. Bradford, *Beloit College*, presiding, consisted of eight contributed papers. These presented striking demonstrations and some further laboratory techniques. In all the program of the Twenty-Third Annual Meeting there were two panel discussions, one round table, three invited papers, twenty-seven contributed papers, the business session, a showing of recent films, and the joint meetings with the American Physical Society. The abstracts of the contributed papers are printed below. The invited papers will probably be published in the *Journal*. It is hoped that either through publication or through the use of the tape recordings the round table and the panel discussions will be made available to many of

those members who were not able to attend the New York meetings.

JAMES M. BRADFORD
Beloit College

Program

Invited Papers

Demonstration of color television. C. N. HOYLER, *Radio Corporation of America.*

What can we expect to learn from high energy accelerators? ENRICO FERMI, *University of Chicago.* (Address of the retiring president of the American Physical Society.)

The metaphysics of a physics teacher. C. N. WALL, *University of Minnesota.* (Response of the Oersted Medalist.)

Summer institute for teachers of physics. J. W. BUCHTA, *University of Minnesota.*

Fellowship program of the National Science Foundation. BOWEN C. DEES, Program Director for Fellowships, *National Science Foundation.*

Richtmyer Memorial Lecture

Fields and particles. JOHN A. WHEELER, *Princeton University.*

Round Tables and Panel Discussions

The functions and mission of AAPT. VERNET E. EATON, *Wesleyan University,* presiding. **Presentation of the problem,** HAROLD K. SHILLING, *Pennsylvania State University.* **What goes into the *American Journal of Physics*,** THOMAS H. OSGOOD, Editor, *Michigan State College.*

Research subsidies and college teaching. KARL S. VAN DYKE, *Wesleyan University,* presiding. **Research in the teaching process.** HERBERT E. LONGNECKER, Dean of the Graduate School, *University of Pittsburgh.* **Report on the Amherst Conference.** THEODORE SOLLER, *Amherst College.* **Balancing research and teaching in the college.** WALTER C. MICHELS, *Bryn Mawr College.* **One approach to the problem of integrating teaching and research.** CHARLES H. SCHAUER, *Research Corporation.*

The responsibility of the physics teacher in engineering education. JOSEPH H. KEENAN, *Massachusetts Institute of Technology,* presiding. L. E. GRINTER, *University of Florida,* President of the American Association for Engineering Education; N. H. FRANK, *Massachusetts Institute of Technology;* L. P. SMITH, *Cornell University;* and H. L. DODGE, *Norwich University.*

Contributed Papers

1. Find the external force. T. D. PHILLIPS, *Marietta College.*—Some experiments were shown which furnish exercise for those who use the word "external" in stating Newton's first law of motion.

2. Derivation of $E=mc^2$ from Maxwell's formula for the pressure of light. GORDON FERRIE HULL, *Dartmouth College.*—Consider a parallel beam of light of one cm² cross section consisting of particles (photons) each of mass m and energy E, striking normally a totally absorbing surface. The energy density is nE, the momentum of each particle is mc, the number striking per second is nc, and the total momentum change per second is nmc^2. By Maxwell's formula this equals nE or $E=mc^2$ for photons. The meaning of this relation, its extension to gross matter, its similarity to Einstein's derivation for the energy of photoelectrons, and to Compton's derivation of the change of wavelength of x-rays due to scattering were shown. Photons have energy, momentum, and gravitation.

3. Teaching automatic digital computers. C. H. DAVIDSON, *University of Wisconsin.*—Automatic computing, one of the new tools of the scientific method, is expanding in scope and application. Training of personnel in understanding, further development, and efficient use is not keeping up. Here is presented a delineation of some of the areas of interest which can be shared by physicists, mathematicians, and engineers in transforming the automatic computer from a gadget to a necessary adjunct of scientific and industrial research. What types of things need to be taught students in the sciences, in mathematics, and in engineering; and where can some of this material be readily obtained?

4. "The ratio of the specific heats" and thermodynamics. L. G. HOXTON, *University of Virginia.*—It is held that Reech's theorem, namely, C_p/C_v equals the ratio of the adiabatic to the isothermal elasticity of the fluid in question, may be deduced in a course before the first law of thermodynamics is taken up. The proof is very brief. This procedure is not only a convenience for the instructor but a help to the student in clarifying his ideas and inculcating the habit of continually keeping the basic principles in sight. A few textbooks use this approach but, strangely, many do not.

5. Some common examples of careless statements in elementary physics textbooks. ERIC RODGERS, *University of Alabama.*—Certain statements commonly found in elementary physics textbooks can easily mislead the student, although the statements themselves are not necessarily incorrect. Some statements of this kind were discussed.

6. A new method for measuring the efficiency of a motor. ROBERT M. WOODS AND ROBERT SHERWIN, *Westminister College.*—The motor to be tested is mounted on a platform which is supported by ball bearings so that it hangs from an axis which coincides with the axis of the motor shaft. An arm attached to this platform is placed on the scale of a balance. A dc generator is mounted on the frame which supports the motor platform. The generator is belted to the motor, and a load resistor and a control rheostat are connected to the generator. The energy can thus be dissipated in the resistor which can be water cooled if necessary. The power output of the motor is computed from the torque required to hold the frame and the speed. The power input can be measured by a wattmeter and/or an ammeter and a voltmeter.

7. A stabilized power supply for the elementary electronics laboratory. CHAS. WILLIAMSON, *Carnegie Institute of Technology.*—This small and compact power supply is set up to furnish the voltages and currents usually required in elementary experiments with electronic circuits. It is stabilized with the aid of two voltage regulator tubes. Input fuses and an output indicator lamp are installed. Wire-wound, three-terminal rheostats of ample overload capacity are used to adjust output voltages. The direct-current output from any one tap is limited to about 20 milliamperes, but two or more separate currents of this magnitude may be taken simultaneously from properly chosen taps. All parts of the circuit are insulated from the chassis and cabinet; thus any point in the circuit may be safely grounded. An unstabilized filament supply is included.

8. Motion pictures of a diffusion cloud chamber. EVAN S. SNYDER AND JOHN J. HEILEMANN, *Ursinus College.*—A diffusion-type cloud chamber of small dimensions was constructed from a plastic cheese box $3\frac{1}{4}$ in. in diameter and $1\frac{1}{2}$ in. deep. The cold surface is a brass plate thermally connected to a cooling mixture of dry ice and alcohol contained in a vacuum bottle. Although the sensitivity changes as the temperature distribution varies with time, the chamber will operate several hours on a few ounces of dry ice, showing the tracks of alpha- and beta-particles. At the time of writing no quantitative measurements have been made, but it seems reasonable to assume that there are tracks from other ionizing particles. Motion pictures were made with an ordinary 16-mm camera at 16 frames per second; these were shown and the chamber was described in detail and exhibited.

9. Transistors in the electronics laboratory. JAMES J. RUDDICK, *Woodstock College.*—After a brief discussion of the purpose of the intermediate laboratory course in electronics, suggestions were made regarding the advisability of introducing one or more experiments on the properties and uses of transistors. Not only do transistors offer an interesting opening into the field of physical electronics but they introduce the student to devices which are bound to have great importance in the instrumentation of basic research. A sample experiment on transistor characteristics is described, as well as the requisite knowledge the student should have in order to profit from the experiment.

10. Problems encountered in routine use of 10-kilocurie gamma radiation source. J. V. NEHEMIAS, *University of Michigan.*—During a year of routine use of a 10-kilocurie gamma radiation source, operational problems have been encountered in control of radiation levels in nearby areas, control of pH and clarity of the water in the source storage well, breakdown of organic plastics under prolonged irradiation, and a discrepancy between nominal and apparent source strength. A short discussion of these problems and the resulting changes in operational procedures plus a brief summary of a typical experiment were presented.

11. Trends in microwave triodes. C. LUTHER ANDREWS, *New York State College for Teachers*, Albany.

12. On repeating Becquerel's experiment. ALFRED ROMER AND BARBARA L. DODDS, *St. Lawrence University.*—Oldenberg suggests Becquerel's historic experiment of exposing a photographic plate wrapped in black paper to the beta rays from uranium as a valuable classroom demonstration.[1] His two-day exposures seem excessive when Becquerel himself could obtain results in five hours,[2] but were confirmed by preliminary experiments with cut film. Becquerel later attributed part of the blackening to secondary rays emitted by the glass backing in his plates,[3] and following this clue we have found with the same emulsion and exposure time stronger effects with plates than cut film. Emulsions of high threshold sensitivity give quicker results than those of inherent high contrast. The faint image produced by short exposure shows better in opaque projection with the plate backed by white paper than when the plate is projected as a transparency. The feasibility of carrying out the demonstration in a single lecture hour will be discussed.

[1] O. Oldenberg, Am. J. Phys. **20**, 111 (1952).
[2] H. Becquerel, Compt. rend. **122**, 501 (1896).
[3] H. Becquerel, Compt. rend. **132**, 734 (1901).

13. A method for the analysis of musical sounds. JAMES M. BRADFORD, *Beloit College.*—A method of sound analysis has been developed whereby the student of an elementary course in physics is able to determine the frequency of the harmonics present in a musical note, as well as to determine the distribution of energy in the harmonic series. To obtain a record the musical note to be studied is sounded before an Astatic Corporation microphone, Model JT30-A, attached to a single input Magnacordette. The note is sounded at as uniform an intensity as possible for a period long enough to fill a five-foot loop of Scotch recording tape. This recording is then played back by the same recording instrument to a Hewlett Packard Model 300 Harmonic Wave Analyzer. Although the musician can play a sustained note with somewhat uniform intensity for only a few seconds the tape loop can be played over and over, thereby enabling the experimenter to obtain values of the overtone frequencies and their average energies in millivolts.

The advantages of the equipment as described: (1) the loops may be recorded in an acoustical room and the tape studied with the analyzer in the laboratory at a later time; (2) the note sounded may be of any frequency—preferably in the range of 300 cps to 600 cps; (3) the tape loop may be played as long as necessary to make the study required; (4) this is an analysis of the wave form produced with the instrument by the person playing the instrument and not a synthesis to obtain a similar wave form; (5) several loops may be analyzed in a laboratory period.

14. Science and society. DONALD J. LOVELL, *Industrial Research Laboratories.*—Science, being basic to technological achievement, plays a role of prime importance in the progress of civilization. It is advocated that an understanding of the limitations and goals of science will help society utilize the products of science. These limitations and goals are outlined and considered in the light of history. The relation of science to culture is exhibited, and a forecast of the future is contemplated.

15. Physicists' privileges and responsibilities in the modern world. LOUIS R. WEBER, *Colorado Agricultural and Mechanical College.*—Recent trends in the development of atomic energy have caused considerable tensions and fears, especially in the American population. Physicists, because of their generally broad academic training and participation in the development of new sources of energy, have an important role in helping to improve understanding not only in their own country but in the other countries of the world. They should therefore seek opportunities, when qualified, to participate in teaching or research assignments outside the United States. Such an experience not only helps to resolve problems in their own minds, but encourages understanding in others.

16. Elementary laboratory instruction in England. SANBORN C. BROWN, *Massachusetts Institute of Technology.*—A firsthand survey was made of elementary laboratory instruction at a number of universities in Great Britain. The British type of experiments, laboratory exams, and teaching techniques will be discussed. English undergraduate laboratory education will be compared with the American system. It will be shown that the English students do many more experiments, and spend well over twice the time in the laboratory than do students in America. The English put a great deal more emphasis on the details of technique, and carry out more experiments on viscosity, surface tension, and thermal properties than is customary in America. Certain generalizations can be drawn from this study which can guide the planning of elementary courses in this country.

17. High school laboratory science. S. W. CRAM, *Kansas State Teachers College.*—Reduced enrollment in the physical sciences both at college and secondary levels causes us to be concerned about ways and means to increase interest on the part of students. Kansas is trying a new course at secondary level called "Laboratory Science" for those who take Biology as a means of escape. The course is a distinct laboratory approach to environmental projects of science concern in the hopes of arousing interest among supposedly nonscience students. Report is to be on reasons for such a course, objectives, and content of course.

18. The need for a revision of the undergraduate physics curriculum. MALCOLM CORRELL, *DePauw University.*—The undergraduate physics curriculum needs review and revision with a view to modernization, streamlining, and the attraction of greater numbers of competent majors. Several ways in which these aims might be advanced are indicated. It is suggested that a committee of the association might be formed as an organized attempt to formulate, evaluate, and propose ways and means for such a curriculum revision.

19. Performance measurement in education. ELMER HUTCHISSON, *Case Institute of Technology.*—Management in industry is coming to depend more and more upon performance measurement and control. Principles have been developed which have wide applicability. In particular they may be applied to teaching in the field of physics and in other fields with considerable success. In this paper specific examples are developed so that a teacher may use them to evaluate his own performance.

20. A philosophy of demonstration experiments. JULIUS SUMNER MILLER, *El Camino College.*—Physics teachers generally consider a demonstration experiment a good one *only if it works*. The function of a demonstration is, indeed, to demonstrate a physical phenomenon, but if it does *only* this—if it works—its usefulness is often substantially delimited. A case is made for the demonstration which is considered "not so good" because its reliability cannot be depended upon. In truth, it is urged that more physics is often taught by a demonstration which fails to work as predicted.

That *a demonstration is for the student* needs pointing up to a goodly number of teachers. Evidence abounds that too many students leave a demonstration not too certain of what happened. The professor too often talks as if the student knew much about the phenomenon already. Consequently he leaves too much unsaid. Worse still, he may talk in a monotone that wants for spirit and drama. In addition, too much of what is done goes on unseen—except by those in the front row.

To sum it up, a demonstration that fails is still good; if guile and deception can be involved, this is good too. But drama and life must be injected and a clear recitation must be given, for, after all, the student sees this for the first time. It is very easy for the professor to leave the discussion obscure since he has toyed with this thing for maybe a dozen years!

21. Some demonstration puzzlers. FLOYD W. PARKER, *Lincoln Memorial University.*—Using such items as a garden hose, a bicycle wheel, and an electric blower, demonstrations were performed whose outcome is at variance with predictions based on intuition rather than basic physical laws. In addition, a modification of the usual Kundt tube laboratory experiment was described.

22. Simple demonstrations. A. D. HUMMEL, *Ball State Teachers College.*—*I. Electrical Oscillator.*—A battery is connected to the primary and a condenser to the secondary of a transformer. An oscilloscope, connected to the condenser, shows the oscillations which are started by opening the primary circuit. They are transient and damped. *II. Experimental Proof of Thevenin's Theorem.*—A wiring diagram and directions are given for verifying the truth of this valuable network theorem. *III. A Short Foucault Pendulum.*—The support is mounted on a table which may be rotated to show that the mounting exerts practically zero torque on the pendulum.

23. Demonstrations with watches on bifilar suspensions. L. G. HOXTON, *University of Virginia.*—The effects of nonrigid supports on the rates of timepieces is well known. Lord Kelvin treated the matter in some detail (1867) in one of his Popular Lectures (Vol. II). These demonstrations, however, seem to possess novel features. In addition to being of interest to students of vibrating systems with

two degrees of freedom and of forced and sympathetic oscillations, they have led to dependable and convenient ways of correcting watches without setting or opening them.

24. Several demonstration experiments. ERIC M. ROGERS, *Princeton University.*—Several demonstration experiments were shown.

25. Orbital magnetization. F. W. WARBURTON, *University of Redlands.*— (Read by title.) Potential energy defined as work one body can do by its force on another includes velocity potentials, leads directly to the generalized Lagrangian equations, and yields a description of magnetization and gyromagnetism as electron orbital motion without spin and with magnetic potential energy equal to the Larmor precession kinetic energy. Two unbalanced gyroscopes, given their precessional kinetic energy by a little push to represent induced electric field causing precession of an electron orbit, have also gravitational potential energy,—$j \cdot m\mathbf{g}$, corresponding to the magnetic potential energy,—$\mathbf{\mu} \cdot \mathbf{B}$. On collision both the kinetic energy of precession and the potential energy are recovered in work done in alignment. In rotation by magnetization the Larmor precession angular momentum is transmitted to the rod by the electrostatic "collision" forces preventing precession, the direct magnetic torque on the orbit not being transmitted to the rod. Precessing gyroscopes mounted on a platform free to rotate illustrate this. Magnetization by rotation is illustrated by balanced gyroscopes made to collide by rotating the platform. The electrostatic "collision" torque causes half as much alignment as the magnetization process where in addition the magnetic torque $\mathbf{\mu} \times \mathbf{B}$ acts directly when precession is prevented.

26. A simple electrical experiment which teaches several lessons. FRANCIS T. WORRELL, *Rensselaer Polytechnic Institute.*—An experiment is described wherein the resistance of a galvanometer is measured by three different methods. Discrepancies are found between measured values. Further experimentation reveals that some assumptions had been made which were unwarranted, and the various measurements can be shown to be consistent with one another.

27. Cooperation between the physics and the engineering departments at the institutional level. ROBERT RESNICK, *University of Pittsburgh.*—Much of the criticism directed at physics courses for engineering students stems from the failure of the physics and engineering departments to communicate directly with one another in regard to such matters. Although each institution has its own peculiar problems, there are many problems common to all. The experience of the University of Pittsburgh Physics Department in meeting these problems by direct communication with each of the engineering departments concerned will be discussed. It is suggested that other institutions might profit from such an approach.

American Association of Physics Teachers

Minutes of the Meeting of the Council Held in New York

The Council of the AAPT met at the American Institute of Physics building on January 28th, 1954. Present at the meeting were: P. E. Klopsteg, President, *National Science Foundation;* M. W. White, President-elect, *Pennsylvania State University;* R. F. Paton, Secretary, *University of Illinois;* F. W. Sears, Treasurer, *Massachusetts Institute of Technology;* T. H. Osgood, Editor, *Michigan State College;* V. L. Bollman, *Occidental College;* V. E. Eaton, *Wesleyan University;* W. C. Michels, *Bryn Mawr College;* C. Williamson, *Western Pennsylvania;* W. Noll, *Kentucky;* R. T. Ellickson, *Oregon;* R. L. Price, *Chicago;* R. R. Meijer, *Chesapeake;* I. Walerstein, *Indiana;* L. R. Weber, *Colorado-Wyoming;* G. Q. Lefler, *Illinois;* J. J. Heilemann, *Eastern Pennsylvania;* W. Geer, *Southern California;* R. R. Palmer, *Wisconsin;* T. D. Phillips, *Appalachian;* F. Verbrugge, *Minnesota;* and S. O. Grimm, *Central Pennsylvania.* The above members represented the Council. Note that every section was officially represented.

Others present to report for committees, or representatives of other societies were: H. A. Barton, AIP; C. J. Overbeck, *Northwestern;* W. Waterfall, AIP; L. Bockstahler, *Northwestern;* R. C. Gibbs, NRC; C. E. Bennett, ASEE; M. H. Trytten, NRC; H. L. Dodge, ASEE; R. M. Sutton, NSTA; J. G. Potter, *University of Texas;* B. B. Watson, AAAS; G. E. C. Kauffman, *South Carolina;* J. W. Buchta, *University of Minnesota;* and C. Hodges, *Temple University.*

The meeting was called to order promptly at 7:00 P.M. by President Klopsteg. The report of the Committee on Nominations was presented by Professor J. G. Potter who also announced the result of the election for the Committee of Tellers.

Declared elected for 1954 were:

President-elect . . . . R. Ronald Palmer
Treasurer . . . . F. W. Sears
Member of Executive Committee
  to 1956 . . . . V. L. Bollman.

This report was accepted by the Council with appreciation for the excellent work done by the Chairman.

The term of Duane Roller as member of the Governing Board of the American Institute of Physics (AIP) representing the AAPT having expired, nominations for this post were requested. M. W. Zemansky submitted the name of Eric Rodgers of *Alabama University.* This nomination was seconded by F. Verbrugge. Other names not being offered, R. R. Palmer moved the nominations be closed. This motion carried unanimously, the council agreeing that Eric Rodgers be the nominee. (At an election held by the Corporation of the AIP on February 20, Eric Rodgers was elected to its Governing Board for the term ending in 1957. The other representatives of AAPT on this board are R. M. Sutton and M. W. Zemansky, 1955, and J. W. Buchta, 1956.)

Professor W. C. Michels submitted the report of the Membership Committee outlining the successful efforts the Committee had made to halt the trend of decreasing

membership which had been observed since the vigorous growth of the society stopped with the retirement of that highly effective previous chairman, Marsh White. There was some discussion of new potential members and it was agreed that further efforts to secure the membership of physicists not in the teaching profession and students of physics eligible for junior membership would be made. Especial emphasis was to be made to secure a more representative membership among physicists in engineering colleges. A commendatory acceptance of this report was moved by M. W. Zemansky, seconded by Marsh White, and voted unanimously.

Professor T. B. Brown, Chairman of the Taylor Memorial Committee, was unable to be present and R. R. Palmer, a member of this committee reported for him. It was brought out that due to extended illness of the chairman not as much as had been anticipated was accomplished during 1953 but an excellent number of suggestions for the proposed manual of advanced experiments in physics had been accumulated. Need for more active participation of the membership at large was definite. This report was received with assurance of continued budgetary support.

Each year the retiring President of AAPT serves as chairman of the Committee on Awards. Professor W. S. Webb having requested release from this responsibility early in 1953, the Executive Committee had appointed M. W. Zemansky to serve as Acting Chairman. Professor Zemansky discussed briefly the difficulties experienced by the committee in making a selection of the Oersted Medalist from among a large group of worthy candidates recommended to this committee each year. The teacher finally agreed upon was Professor C. N. Wall of the *University of Minnesota*. W. C. Michels moved, seconded by V. E. Eaton, that the council approve this action of the committee, and this part of the report was voted unanimously.

For Honorary Membership in the Association the name of Detlev W. Bronk was submitted. On motion by V. E. Eaton, seconded by W. C. Michels, this was approved unanimously by the Council.

The names of T. D. Cope, P. Kirkpatrick, C. J. Overbeck, and M. H. Trytten were proposed for special citation by the Committee and the Council instructed the Chairman to submit these names for action at the annual business meeting of the Association scheduled for Saturday morning, January 30. (See the report of business meeting below.)

The new films to be shown at the Saturday afternoon meeting were described very briefly by the Chairman of the Visual Aids Committee, M. W. Zemansky. He reported that five new films had been prepared during the year under immediate supervision of committee members. Criticisms of these films and suggestions for new ones and improvement of old ones were solicited. It was generally agreed that the work of this committee was an outstanding contribution to the teaching of physics.

A short printed report of the activities of the Committee on Engineering Education prepared by the Chairman, J. H. Keenan, was distributed. President Klopsteg called attention to the fine success of this committee in planning and arranging for Panel Discussion given in the afternoon.

C. E. Bennett and J. G. Potter both spoke on the objectives of this committee and stressed the need for increased awareness of physics teachers of the problems arising from the need for appropriately training increasing numbers of physicists serving industry and research institutions. Conferences between engineering educators and teachers of physics are being actively encouraged by the National Science Foundation. T. H. Osgood, Editor of the *American Journal of Physics*, offered to work out a system of mutual reports with the *Engineering Journal* and it was agreed that the committee would be alert to this possibility.

The Committee on Letters, Symbols, and Abbreviations sent reprints of its published report which were distributed. Duane Roller, the chairman, recently appointed editor of *Science*, was unable to be present. He submitted a request to change the name of this committee to Terminology, Letter Symbols, and Abbreviations. The Council approved this. It is expected that this committee will cooperate actively in 1954 with a similar committee being planned by the American Physical Society and that the Joint Committee will be active on an international level.

A report summarizing the activities of the Program Committee was presented by President-elect M. W. White, Chairman ex officio of this committee in charge of plans and programs of society meetings. Active cooperation of R. R. Palmer, in charge of plans for the current meeting, was expressed. On behalf of the *Pennsylvania State University*, Professor White invited the Association to hold its June meeting in 1955 at State College, Pennsylvania, in conjunction with the national meeting of the American Society of Engineering Education (ASEE). This meeting would be timed to coincide with the 100th anniversary of the founding of the University. Motion to accept this invitation was made by R. R. Palmer, seconded, and carried with enthusiasm.

Preparation of College Teachers of Physics was discussed by J. W. Buchta, Chairman of the Association's Committee studying problems in this field. Active programs of symposium and workshop nature carried on at the *University of Minnesota* were reviewed by Professor Buchta. Professor C. J. Overbeck of *Northwestern University* requested cooperation in plans for a June meeting at *Northwestern University* designed to promote exchange of ideas between experienced teachers of physics and young graduate students in physics just beginning their careers as teachers.

The Special Committee on Contracts with the AIP was discontinued after reporting that the previous agreement would be continued.

At a preliminary meeting of the Executive Committee of the Association held the evening of January 27, considerable time had been spent discussing possible solutions for the serious and continued annual deficit. The treasurer, F. W. Sears, pointed out in reporting on the situation to the Council that the cash balance would be used up by the end of 1954 unless the activities of the Society were drastically curtailed. No one on the Council thought this should be done and it was accordingly voted to submit an amendment to the by-laws to the members at the annual business meeting scheduled for January 30. This amend-

ment in effect would increase the annual dues of regular members to not more than $8.00. Dues of Junior members were to remain at $4.00. The exact annual rate for 1955 was to be determined by the Executive Committee as soon as a better estimate of annual expenses could be made. It appeared that $7.00 would be sufficient to cover expenses as envisioned at the time of meeting. The Secretary was instructed to prepare the amendment for presentation at the annual business meeting. The estimated budget presented by the Treasurer, F. W. Sears, was approved by unanimous vote of the Council.

President Klopsteg reported that Editor Osgood had agreed to serve the Association in that capacity for another term of three years. This announcement together with the Editor's report received pleased, appreciative, and unanimous approval by the Council. A charge of $3.00 for separate copies of the 20 year cumulative index was also voted.

The Council approved the appointment of Richard A. Beth, *Western Reserve University*, J. D. Stranathan, *University of Kansas*, and Hugh C. Wolfe, *Cooper Union*, as Associate Editors for the period 1954–56 in place of W. W. McCormick, *University of Michigan*, C. W. Ufford, *University of Pennsylvania*, and George H. Vineyard, *University of Missouri*, who completed their terms of service in 1953.

P. E. Klopsteg and M. W. White reported briefly on their activities as representatives of AAPT in AAAS. The Council was pleased to note that Dr. Klopsteg was a member of the Governing Board of AAAS and that Duane Roller had recently been appointed Editor of *Science*. It was felt that the Association was fortunate to be so adequately represented in the AAAS.

Dr. B. B. Watson distributed reprints of the report of the Cooperative Committee of the AAAS on science and mathematics teaching in the secondary schools appearing in *Science*, October 30, 1953. Dr. Watson has represented the AAPT on this important committee for several years.

Dr. M. H. Trytten reported on his work with a subcommittee of the American Council on Education (ACE). The ACE had raised its annual dues to $200 in 1953. The AIP had supported the AAPT by rebating this fee to the Association but their Executive Committee voted to drop this membership and the Executive Committee of AAPT decided that mutual advantage of this membership did not justify this new expense especially in the light of the Association's continuing deficit. The Council voted to sustain the Executive Committee in this decision and the Secretary was instructed to so inform the ACE.

Dr. R. M. Sutton, representing the Association on the National Science Teachers Association, reported that this organization was an active one and growing in significant influence in the field of science teaching at the secondary level.

President Klopsteg then called for reports from the representatives of the fourteen regional sections of AAPT. Each had prepared a written report for distribution to all the Council members and spoke briefly on local activities for 1953 and plans for 1954.

The Wisconsin section submitted a suggested amend-ment to the Constitution of AAPT. After suggesting a few minor changes, the Council voted to instruct the Secretary to conduct a mail ballot of the entire membership to determine whether the amendment should be adopted. The Council also voted to request each section having special business requiring action of the Council at annual meetings to distribute this information to the members of the Council in advance of the meeting.

There being no further business to bring up, the Council was declared adjourned at 11:05 P.M.

Annual Business Meeting
Harkness Theatre, January 30, 1954

President Klopsteg called the meeting to order and introduced the President for 1954, M. W. White, who presided on behalf of the retiring president who had to leave for another meeting.

The Secretary reported that as a result of the annual ballot the members had chosen for 1954 R. R. Palmer, President-elect, F. W. Sears, Treasurer, and V. L. Bollman, member of the Executive Committee.

On motion of the chairman of the Committee on Awards, seconded from the floor, Dr. Detlev W. Bronk was unanimously elected to Honorary Membership in the AAPT. For noteworthy service to the Association, T. D. Cope, P. Kirkpatrick, C. J. Overbeck, and M. H. Trytten were voted Special Citations.

President White pointed out the need for increased services to the membership from the national organization and the need for increased income to cover the current expenses. The Secretary moved that the first sentence of Sect. 1, Art. 1, of the By-laws be amended to read: "The annual dues of regular members shall be not more than $8.00 and of junior members $4.00." It was pointed out that the Executive Committee for 1954 had been instructed by the Council to set the dues for 1955 at a later time in the year when the expenses for the year could be estimated more accurately and that the annual dues for 1955 would probably be set at only $7.00. After further discussion and answering of questions the amendment to the By-laws was adopted with no dissenting votes.

President-elect R. R. Palmer representing the Wisconsin Section of AAPT presented a proposed amendment to Art. IX of the Constitution of AAPT and moved that the Secretary be instructed to conduct, on behalf of the Council, a mail ballot of the entire membership for its adoption or rejection. If this amendment is adopted, Art. IX will read: "This constitution may be amended by a two-thirds majority of those voting in a mail ballot of all the members. This ballot shall be conducted by the Council. Proposed amendments, sponsored either by a majority of the Council or by a petition to the president signed by at least two percent of the total number of members in the latest published membership list, must be submitted to all the voting members for a vote within one year after the annual meeting following its presentation." This motion was seconded and carried.

Professor J. W. Buchta, having explained the solar eclipse "arranged" for the benefit of the Association for

the morning of June 30 by an enthusiastic committee at the *University of Minnesota*, urged all members to attend the meeting at Minneapolis, June 28, 29, and 30.

A unanimous vote of appreciation for the excellent facilities provided for the meeting and to the local committees who had planned so effectively was voted with enthusiastic applause.

The AAPT was organized in 1930. Many of its founders and loyal workers over these years are now deceased. The names of several of the more recently deceased were mentioned from the floor. Among them were the names of O. H. Blackwood, R. A. Loring, and W. R. Wright. To such as these the AAPT owes the debt of carrying on; a rising vote of respect honoring them was taken. Following this vote the 1954 Business Meeting of AAPT was adjourned.

R. F. PATON
Secretary

Report of the Treasurer

Statement of Cash Receipts and Expenditures

For the Year Ended December 31, 1953

Cash on hand January 1, 1953		$9 818.53
RECEIPTS		
Dues received from American Institute of Physics	$15 839.09	
Royalties	384.14	
Interest on United States bonds	300.00	
Reimbursement from American Society of Engineering Education for travel expense	40.00	
Reimbursement for American Council on Education dues	200.00	
Miscellaneous	6.00	
Total receipts		16 769.23
		26 587.76
EXPENDITURES		
Publication of *American Journal of Physics*	9 825.24	
Editor's office	1 933.21	
Assistant Editor	900.00	
Cumulative Index	1 031.30	
American Institute of Physics		
10 percent of dues	1 597.10	
Collection of dues	804.86	
Miscellaneous	438.29	
President's office	57.60	
Secretary and Treasurer	691.98	
Membership Committee	331.60	
Taylor Memorial	591.69	
Committee on Engineering Education	426.65	
Travel—American Association for Advancement of Science	153.40	

Richtmyer Lecture	100.00	
Program Committee	23.04	
American Council on Education dues	200.00	
Report—Iowa meeting	395.50	
Honorary membership certificates	23.00	
Miscellaneous	74.50	
Total expenditures		19 598.96
Cash on hand December 31, 1953		$6 988.80

The Association owned on December 31, 1953, $15 000 par value United States Government 2 percent Treasury Bonds, due September 15, 1953 to December 15, 1954.

FRANCIS W. SEARS
Treasurer

HARRIS, KERR, FORSTER & COMPANY
ACCOUNTANTS AND AUDITORS

The Officers and Members,
American Association of Physics Teachers.

Gentlemen:

We have examined the accompanying statement of cash receipts and expenditures of the American Association of Physics Teachers, Francis W. Sears, Treasurer, for the year ended December 31, 1953.

We have verified the cash balance at December 31, 1953, by reconciliation with a certificate obtained directly from the depository, and the United States Bonds by inspection at Box 742 in the Kendall Square Branch of the Harvard Trust Company, Cambridge, Massachusetts.

We have traced all cash shown to have been received to the bank statements and have verified all payments made during the year by reference to paid checks and to other acceptable vouchers.

In our opinion, the accompanying statement of cash receipts and expenditures fairly summarizes the reported cash transactions of the Association for the year ended December 31, 1953.

Respectfully submitted,
(s) Harris, Kerr, Forster & Company

American Journal of Physics

Report of the Editor for the Year 1953

By far the most important activity of the editorial office during 1953 was the preparation and completion of a twenty-year cumulative index of the *Journal* covering the years 1933 to 1952. Our subscribers received this cumulative index as Part II of the December, 1953, issue. I am pleased to record the generous support for this project that was received from the National Science Foundation. Its contribution covered approximately sixty percent of the total cost, the remainder being provided by the Association itself.

The twenty-year cumulative index is almost entirely the work of the Assistant Editor, Dr. B. H. Dickinson, to whom we owe a debt of gratitude. He pushed the work

vigorously during the early months of 1953 so that the manuscript was available to the printer six or seven months before the expected publication date. We hope that the index will prove useful to our members, enabling them to profit as much from back numbers as from current issues.

Three Associate Editors, after three years of meritorious service, came to the ends of their terms of office on December 31—W. W. McCormick, *University of Michigan;* C. W. Ufford, *University of Pennsylvania;* George H. Vineyard, *University of Missouri.* To fill these places I recommend the appointment of Richard A. Beth, *Western Reserve University,* J. D. Stranathan, *University of Kansas,* and Hugh C. Wolfe, *Cooper Union,* for the three-year period 1954–1956 inclusive. New Associate Editors do not usually come to their task without previous experience, for the Editor is likely to have called upon them for advice already.

Only one new feature was introduced in the *Journal* pages during the year—a section on Practical Aids for Physics Teachers. It has been a source of disappointment to the editors that this section has been very poorly supported. The number of contributions has not been sufficient to fill the desired space and many of the items have been supplied from the editor's own files. If further support and interest are not manifested in the next few months, this section will be dropped. Apparently there is no real need for it.

A short summary is given below of the relationship between articles submitted and articles published in the *Journal* for the period September, 1952, to May, 1953, inclusive. We hope that it will answer questions that have been raised frequently by members of the Association:

September 1952 to May 1953 Inclusive

	Regular Section	Notes and Letters
No. of articles published	93	77
No. of *Journal* pages published	547	52
No. of articles rejected	16	18
No. of *Journal* pages rejected	99	17

Material rejected included some that was obviously pseudoscience. The numbers of pages of rejected material are inevitably approximations.

Ten articles aggregating 104 *Journal* pages were contributed to the Regular Section on direct invitation or request of the Editor.

The Editor and Assistant Editor wish to place on record their great debt to the unselfish work of the group of nine Associate Editors. Without them the editing of the *Journal* would have been impossible. They also express their thanks to the staff of the Publication Office in New York and especially to Miss Ruth Bryans, whose work has been accurate and on schedule.

The Editor appreciates the friendly cooperation that he has received from authors, associate editors, and the American Institute of Physics.

THOMAS H. OSGOOD
Editor

Report of the Committee on Awards

The Committee on Awards met in Pittsburgh during the Summer Meeting of 1953 and, after a thorough discussion, cast a preliminary ballot. A mail vote was taken in July, and another in August. The results of these considerations are contained in the following recommendations which were moved and carried unanimously at the Meeting of the Executive Council at New York on January 28th and also at the Business Meeting of the members on January 30th, 1954.

Honorary Membership

The Committee on Awards recommends to the Executive Council that honorary membership be conferred upon Dr. DETLEV BRONK, former president of *Johns Hopkins University*, President of the National Research Council, and Head of the Rockefeller Institute for Medical Research. Dr. Bronk has served tirelessly on many committees and groups concerned with the teaching of science in the United States and is an outstanding champion in the struggle to improve the teaching of science in colleges and universities.

Citations

The Committee on Awards recommends to the Executive Council that citations be awarded to the following men:

(1) T. D. COPE, in recognition of his contributions to teaching, both through his classroom work and through his healthy criticism of physics textbooks, and in recognition of his tireless efforts, as the second secretary of our Association in the period from 1937 to 1943, to advance the cause of physics teaching and to increase the strength of AAPT.

(2) P. KIRKPATRICK, in recognition of the fact that he has been able to combine an outstanding research program leading to over fifty published papers with an arduous teaching schedule, and in recognition of his loyal service to our Association as vice-president in 1946, president in 1947, and chairman of the Constitution Committee in 1950.

(3) C. J. OVERBECK, in recognition of outstanding success as a teacher, of his loyal devotion to our Association as secretary in the period from 1943 to 1949, and of his willingness at all times to undertake every kind of service to further the aims of AAPT.

(4) M. H. TRYTTEN, in recognition of his long period of service in the cause of physics teaching as representative of our Association on many committees in Washington, as technical aid in the Office of Scientific Research and Development, and in the War Manpower Commission as Chairman of the Selective Service Advisory Committee which recommended the adoption of the current program for deferment of college students, and as Director of the Office of Scientific Personnel of the National Research Council.

The Committee on Awards
P. E. KLOPSTEG
R. F. PATON
R. M. SUTTON
W. S. WEBB (*Chairman*)
M. W. ZEMANSKY.

The American Institute of Physics

INCORPORATED

57 East 55 Street   New York 22, N.Y.

MEMBER SOCIETIES:

American Physical Society
KARL K. DARROW, *Secretary*, Columbia University, New York 27, New York.

Optical Society of America
ARTHUR C. HARDY, *Secretary*, Massachusetts Institute of Technology, Cambridge 39, Massachusetts.

Acoustical Society of America
WALLACE WATERFALL, *Secretary*, 57 East 55th Street, New York 22, New York.

Society of Rheology
W. R. WILLETS, *Secretary*, Titanium Pigment Corporation, 99 Hudson Street, New York 13, New York.

American Association of Physics Teachers
R. F. PATON, *Secretary*, University of Illinois, Urbana, Illinois.

AFFILIATED SOCIETIES:

American Crystallographic Association
American Society for Metals
Electron Microscope Society of America
Physical Society of Pittsburgh
Physics Club of Chicago
Physics Club of Philadelphia
Cleveland Physics Society
Physics Club of the Lehigh Valley
Sigma Pi Sigma

ASSOCIATES:

Over seventy-five leading industrial enterprises and other institutions which contribute to the support of the Institute.

JOURNALS:

The Physical Review
S. A. GOUDSMIT, *Editor*, Brookhaven National Laboratory, Upton, Long Island, New York.

Reviews of Modern Physics
J. W. BUCHTA, *Editor*, University of Minnesota, Minneapolis, Minnesota.

Journal of the Optical Society of America
WALLACE R. BRODE, *Editor*, National Bureau of Standards, Washington 25, D. C.

The Journal of the Acoustical Society of America
FLOYD A. FIRESTONE, *Editor*, 57 East 55th Street, New York 22, New York.

American Journal of Physics
THOMAS H. OSGOOD, *Editor*, School of Graduate Studies, Michigan State College, East Lansing, Michigan.

The Review of Scientific Instruments
J. B. HORNER KUPER, *Editor*, Brookhaven National Laboratory, Upton, Long Island, New York.

The Journal of Chemical Physics
CLYDE A. HUTCHISON, JR., *Editor*, Institute for Nuclear Studies, University of Chicago, Chicago 37, Illinois.

Journal of Applied Physics
ROBERT L. SPROULL, *Editor*, Rockefeller Hall, Cornell University, Ithaca, New York.

Physics Today
ROBERT R. DAVIS, *Editor*, American Institute of Physics, 57 East 55 Street, New York 22, New York.

THE American Institute of Physics was founded in 1931 as a federation of leading societies in the field of Physics. It combines into one operating agency those functions on behalf of physics which can best be done by the Societies jointly. Its purpose is the advancement and diffusion of physics and its applications to human welfare. To this end it publishes for itself or the Societies the nine journals listed on this page; promotes unity and effectiveness of effort among all who are interested in physics; renders numerous direct services to physicists and to the public; and cooperates with government agencies, national associations, educational institutions, technical industries, and others in such manner as to realize the opportunities and fulfill the responsibilities of physics as an important and constructive human activity.

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